Quantum Properties of Neutrons

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Particle-Wave Properties

Remarks on Neutron Interferometry

Quantum State Preparation and Measurement

Magnetic Noise Dephasing and Decoherencing

Contextuality

Topological Phases

Ultracold Neutrons and Phase Space Transformation

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The Neutron

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Particle Properties

$m = 1.674928\,(1) \times 10^{-27}$ kg

$s = \frac{1}{2}\hbar$

$\mu = -9.6491783(18) \times 10^{-27}$ J/T

$\tau = 887(2)$ s

$R = 0.7$ fm

$\alpha = 12.0\,(2.5) \times 10^{-4}$ fm^3

u - d - d - quark structure

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m.v}$$

Schrödinger

$$H\psi(\vec{r},t) = i\hbar\frac{\delta\psi(\vec{r},t)}{\delta\ t}$$

&

boundary conditions

Wave Properties

$$\lambda_c = \frac{h}{m.c} = 1.319695\,(20) \times 10^{-15} \text{ m}$$

For thermal neutrons
= 1.8 Å, 2200 m/s

$$\lambda_B = \frac{h}{m.v} = 1.8 \times 10^{-10} \text{ m}$$

$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8} \text{ m}$$

$$\Delta_p = v.\Delta t \cong 10^{-2} \text{ m}$$

$$\Delta_d = v.\tau = 1.942(5) \times 10^{6} \text{ m}$$

$$0 \le \chi \le 2\pi\ (4\pi)$$

m ... mass, s ... spin, μ ... magnetic moment, τ ... β-decay lifetime, R ... (magnetic) confinement radius, α ... electric polarizability; all other measured quantities like electric charge, magnetic monopole and electric dipole moment are compatible with zero

$-\mu B$ ________________

↕ two level system

μB ________________

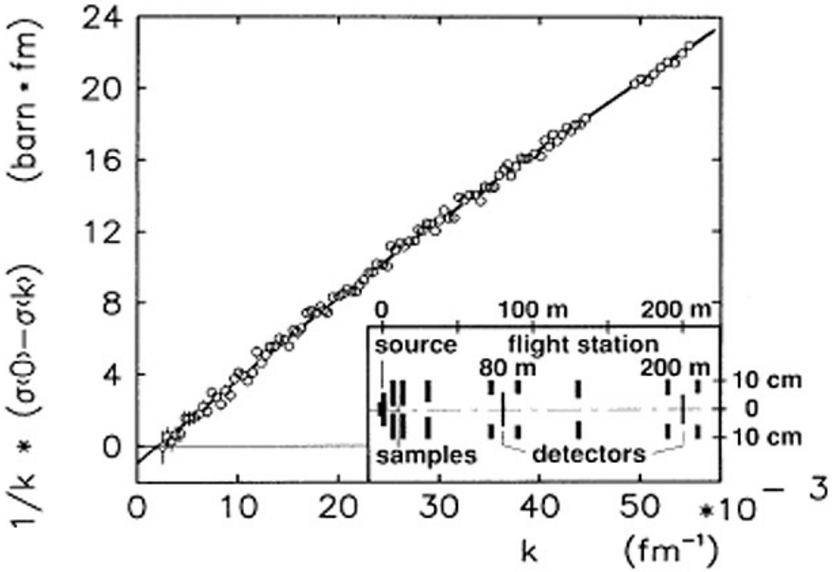
λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk.... momentum width, Δt ... chopper opening time, v ... group velocity, χ ... ... phase.

Electic Polarizability

$$\vec{d} = \alpha \vec{E} \qquad V = -\frac{1}{2}\vec{d}\vec{E} = -\frac{1}{2}\alpha E^2$$

$$\sigma_s(k) = \sigma_s(0) + a \cdot k + b \cdot k^2 + O(k^4)$$

$\Rightarrow$ **Coulomb field of Pb-208**



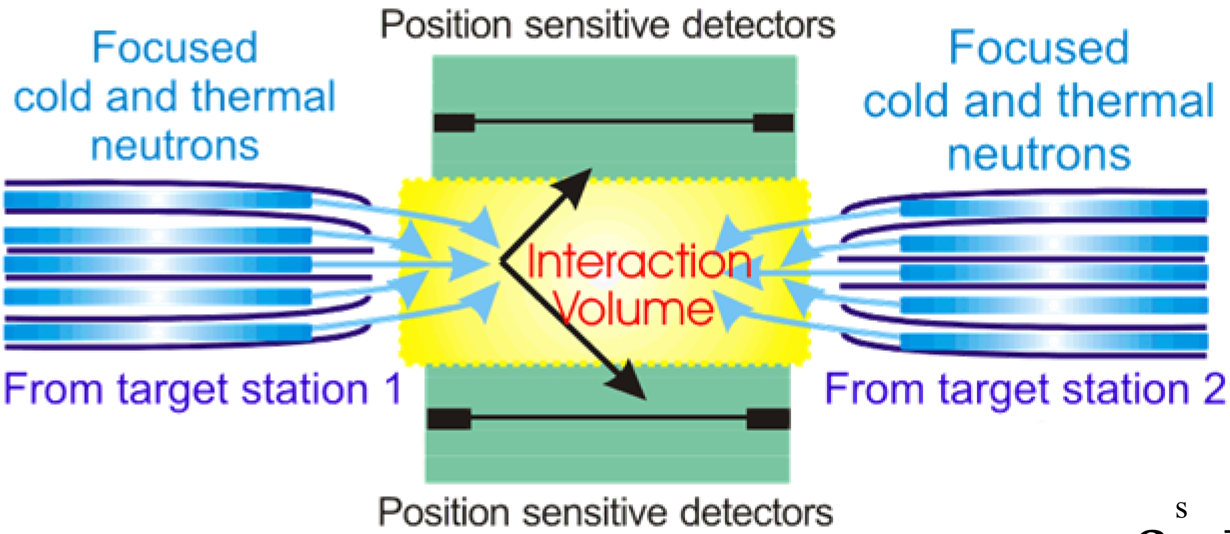
$=(1.20 \pm 0.15 \pm 0.20) \times 10^{-3}$ fm^3

J. Schmiedmayer, P. Riehs, J.A. Harvey, N. W. Hill, Phys. Rev. Lett. 66 (1991) p.1015

Charge dependence and charge symmetry

$$n + n \rightarrow n + n$$

$$a_{np}^{s} \approx a_{pp}^{s} \approx a_{nn}^{s} \quad ?$$



$$a_{nn}^{t} \equiv 0 \quad ?$$

Experiment:

$$a_{np}^{s} = -23.678(28)\,\text{fm}$$

$$a_{pp}^{s} = -17.3(1.0)\,\text{fm}$$

$$a_{nn}^{s} = -16{,}8(1{,}3)\,\text{fm}$$

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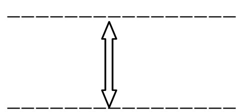
$-\mu B$



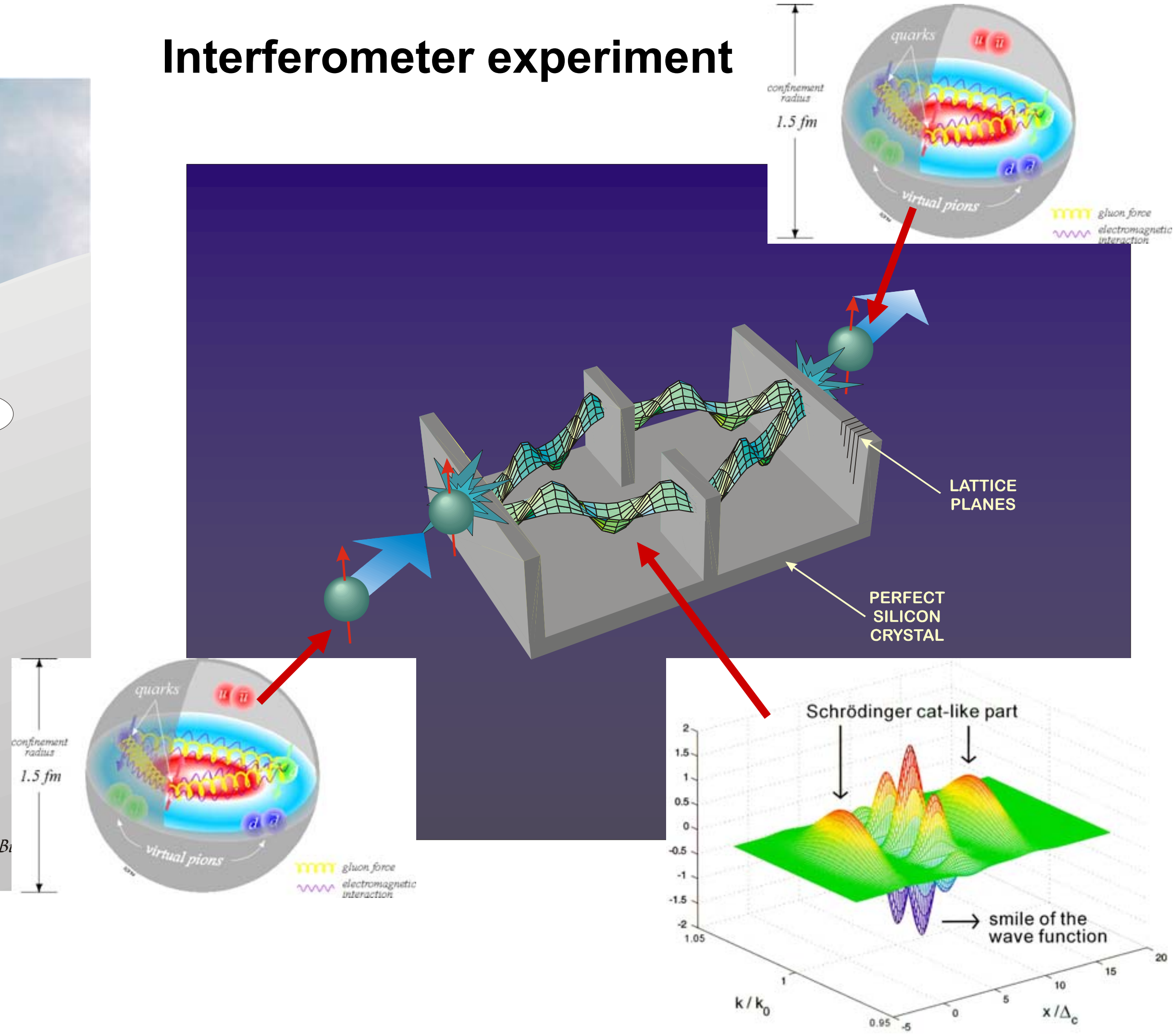
μB

two level system

λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk.... momentum width, Δt ... chopper opening time, v ... group velocity, χ ... ... phase.

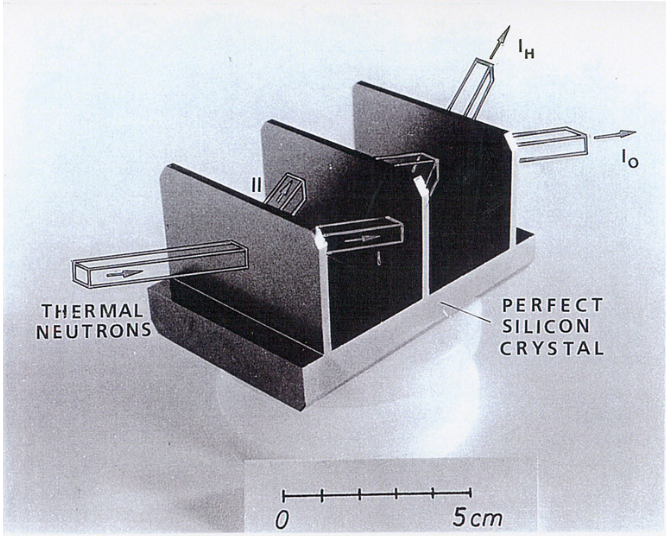
Quantum skier

Interferometer experiment



$$I_0 \;\propto\; \left|\psi_0^{I} + \psi_0^{II}\right|^2 \;\propto\; A + B\cos\chi$$

$$\chi = \oint \vec{k}\,d\vec{s} = (1-n)kD_{eff} \;\equiv\; -Nb_c\lambda D_{eff} = \Delta\cdot k = \Delta k \cdot D_{eff}$$

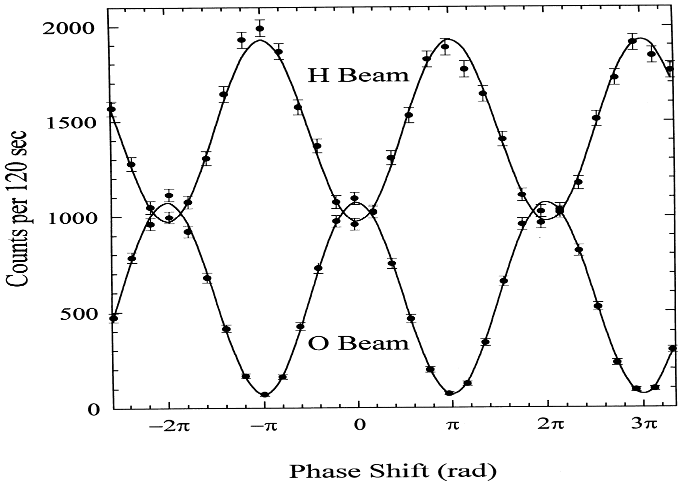


Self interference

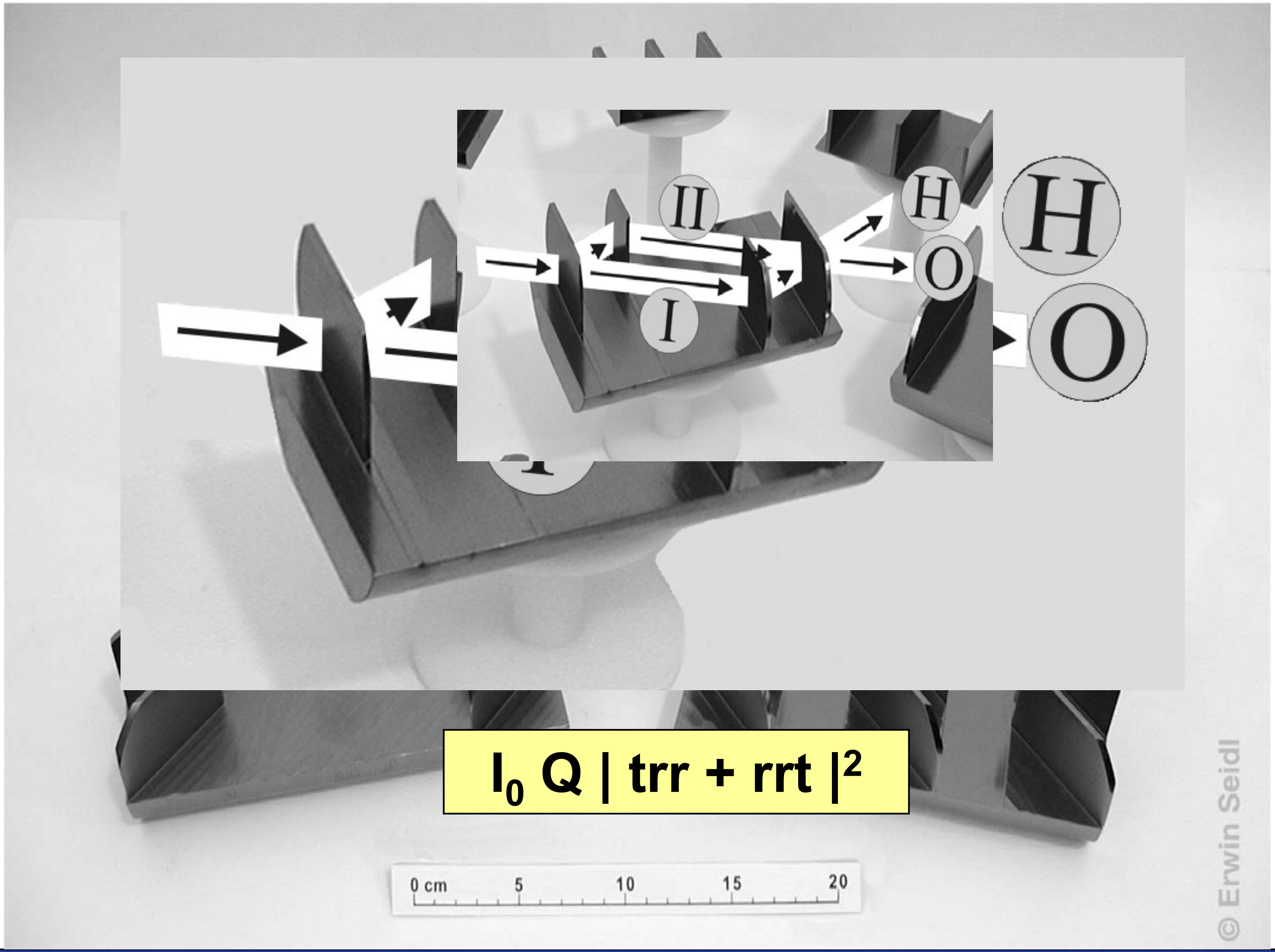
(phase space density ~10^{-14})

Efficiency of detectors, polarizers, flippers >99%

H. Rauch, W. Treimer, U. Bonse, Phys.Lett. A47 (1974) 369

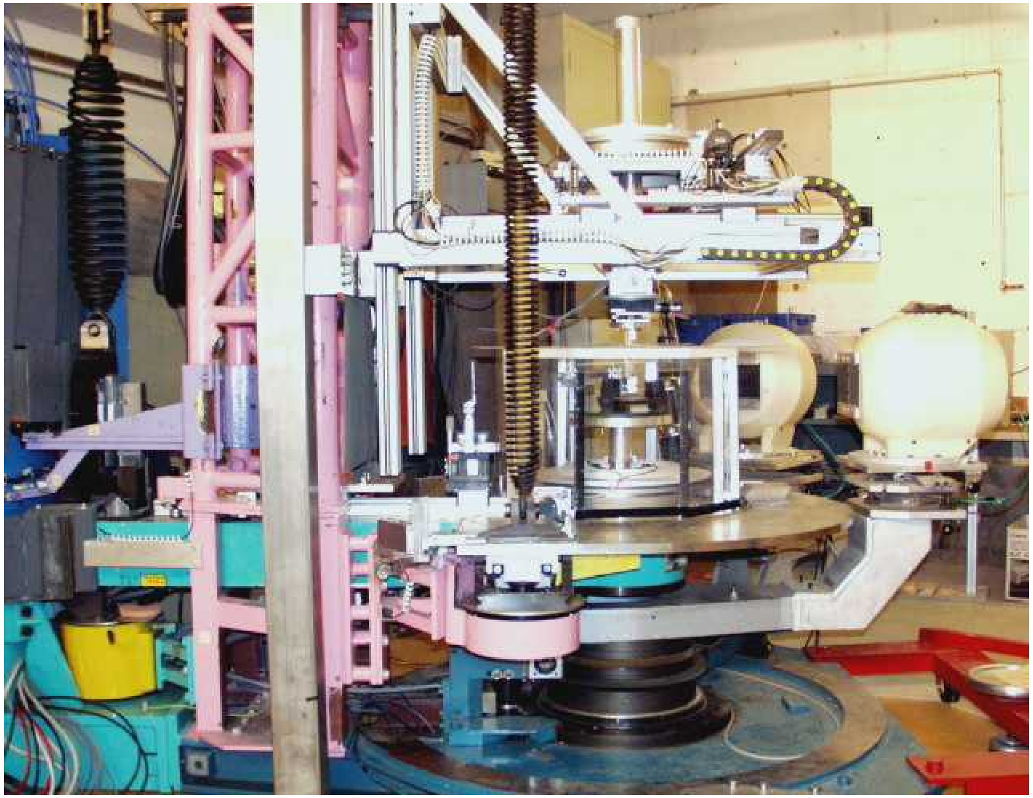


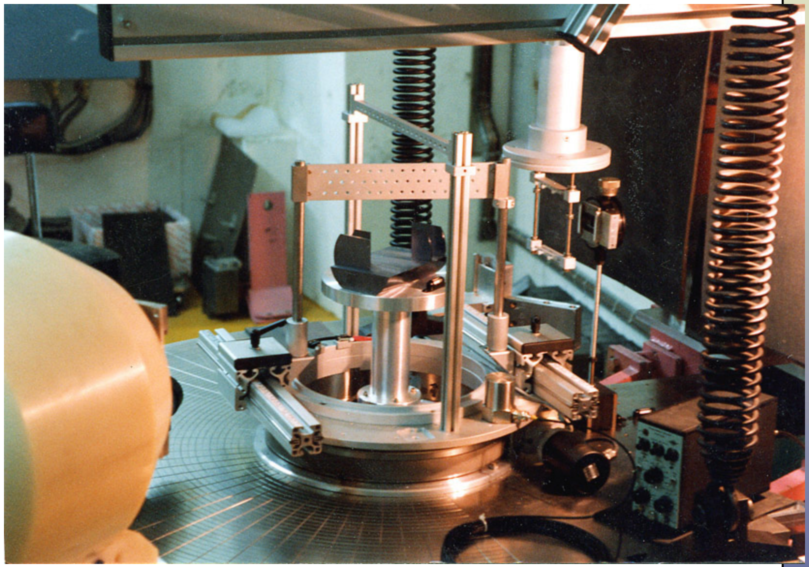
Interferometer family



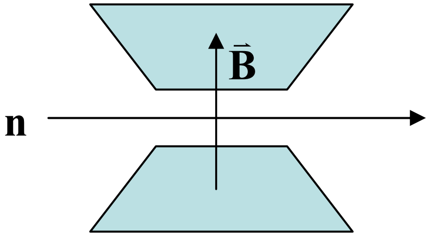
$I_0 \, Q \, | \, trr + rrt \, |^2$

NEUTRON INTERFEROMETER SET-UP S18
AT ILL GRENOBLE





Spinor Symmetry



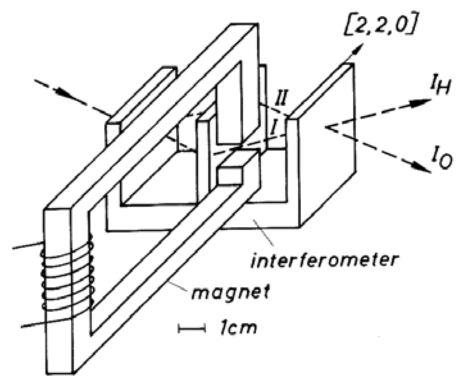
$$\psi^{II} \rightarrow \psi^{I}\, e^{-iHt/\hbar} = \psi^{I}\, e^{-i\vec{\mu}\vec{B}t/\hbar}$$

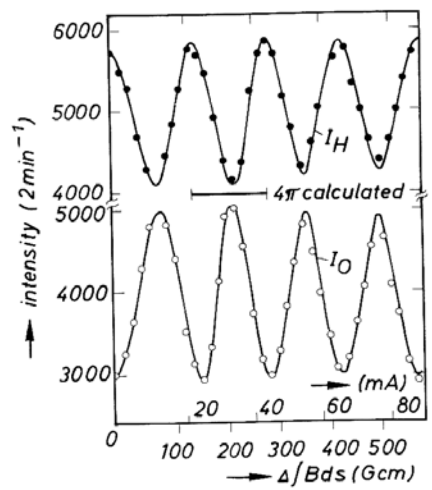
$$= \psi^{I}\, e^{-i\mu\vec{\sigma}\vec{B}t/\hbar} = \psi^{I} e^{-i\vec{\sigma}\vec{\alpha}/2}$$

$$|\alpha| = \frac{2\mu Bt}{\hbar} = gBt \cong \frac{2\mu B}{\hbar}\frac{\ell}{v}$$ ... Larmor angle

$$\psi(\alpha) = \begin{pmatrix} e^{-\alpha/2} & 0 \\ 0 & e^{\alpha/2} \end{pmatrix} \begin{pmatrix} \psi^{I}_{\uparrow}(0) \\ \psi^{I}_{\downarrow}(0) \end{pmatrix}$$

Theory: H.J.Bernstein, Phys.Rev.Lett. 18(1967)1102,
Y.Aharonov, L.Susskind, Phys.Rev. 158(1967)1237





$$\psi(2\pi) = -\psi(0)$$
$$\psi(4\pi) = \psi(0)$$

$$I_0 \propto \left|\psi^{I}_{0}(0) + \psi^{I}_{0}(\alpha)\right|^2 = \frac{I_0(0)}{2}\left(1 + \cos\frac{\alpha}{2}\right)$$

Experiment: H.Rauch, A.Zeilinger, G.Badurek, A.Wilfing, W.Bauspiess, U.Bonse, Phys.Lett. 54A(1975)425
S.A.Werner, R.Colella, A.W.Overhauser, C.F.Eagen, Phys.Rev.Lett. 35(1975)1053
A.G.Klein, G.I.Opat, Phys.Rev. D11(1976)523
E.Klempt, Phys.Rev. D13(1975)3125
M.E.Stoll, E.K.Wolff, M.Mehring, Phys.Rev. A17(1978)1561

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GRAVITATION

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$$H = \frac{p^2}{2m_i} - G\frac{Mm_g}{r} - \vec{\omega}\vec{L} \cong \frac{p^2}{2m_i} + m_g g z - \vec{\omega}\vec{L} + V_0$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t - \frac{1}{2}\vec{g}t^2 + \frac{1}{3}t^3\vec{\omega}\times\vec{g}$$

$$\beta = \oint \vec{k}d\vec{r} = \frac{m_i}{\hbar}\oint \dot{\vec{r}}d\vec{r} = -2\pi m_i m_g \frac{Ag}{h^2}\lambda_0 \sin\phi + \frac{4\pi m_i}{h}\vec{\omega}\times\vec{A}$$

A ... area enclosed by the coherent beams

ϕ ... colatude angle

$$\beta = -q_{grav}\sin\phi + q_s \sin\varepsilon$$

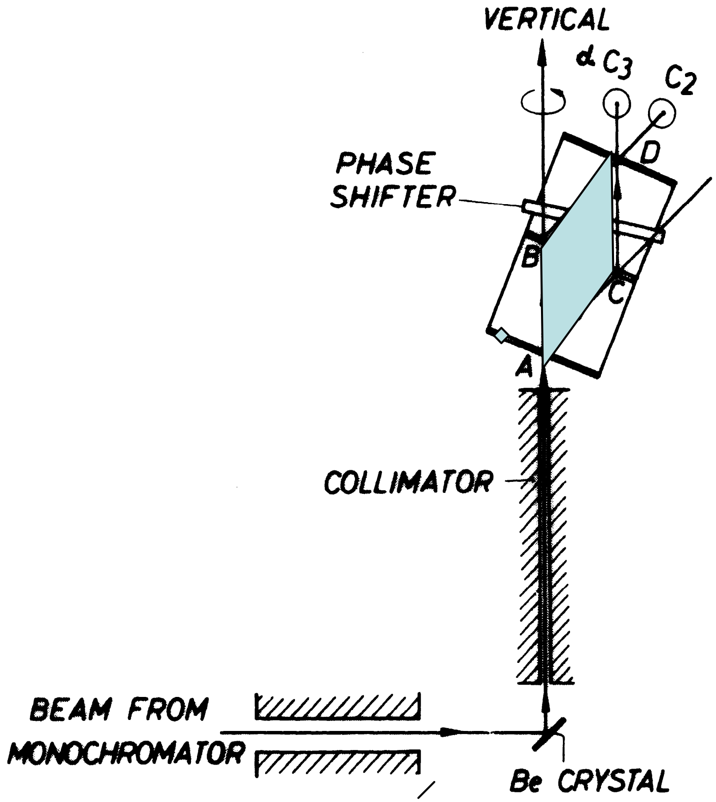
$$2\pi m_i m_g g\lambda A / h^2$$

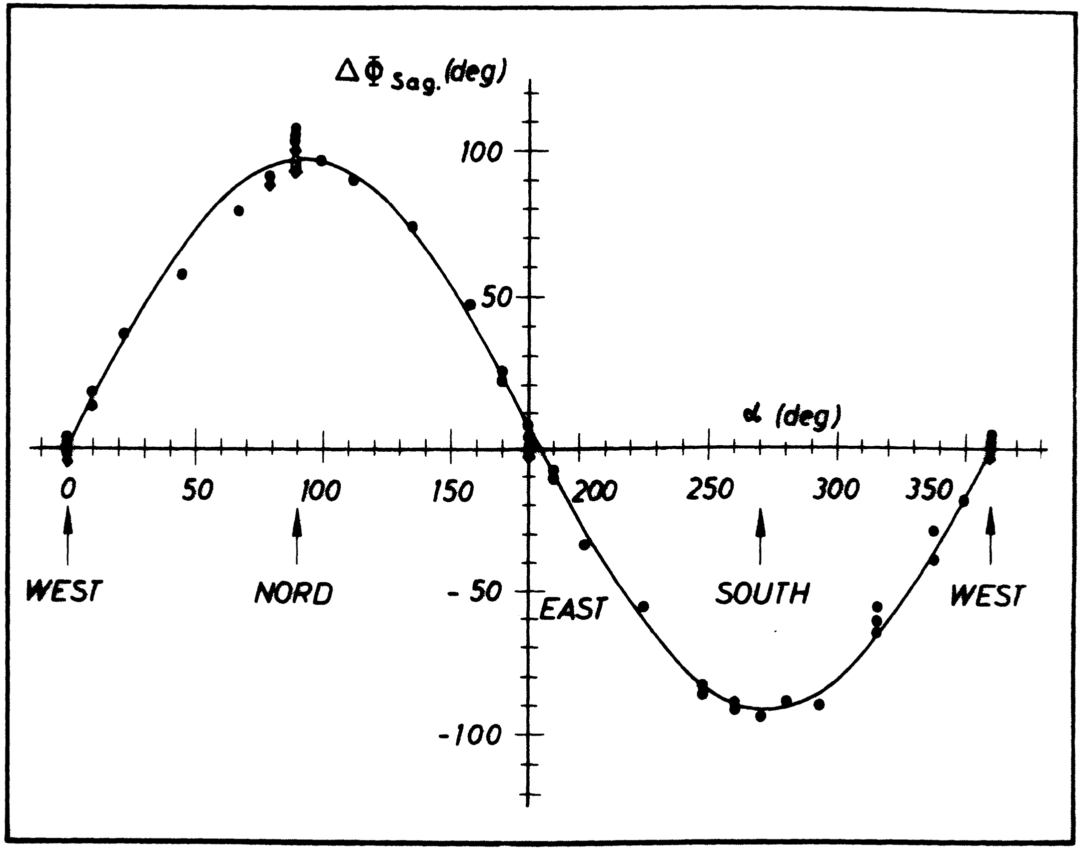
$$4\pi m_i \omega A \sin\phi / h$$

- **EXPERIMENT:**
- Colella, Overhauseer,Werner, 1975
- Werner, Staudenmann, Collela, Overhauser, 1975, 1978
- Bonse & Wroblewski 1983
- Atwood, Shull, Arthur 1984

- **THEORY:**
- Page 1975;
- Anandan 1977
- Stodolsky 1979
- Audretsch & Lommerzahl 1982

Earth Rotation Effect





J.L.Staudenmann, S.A.Werner, R.Colella, A.W.Overhauser, PR/A21(1980)1419

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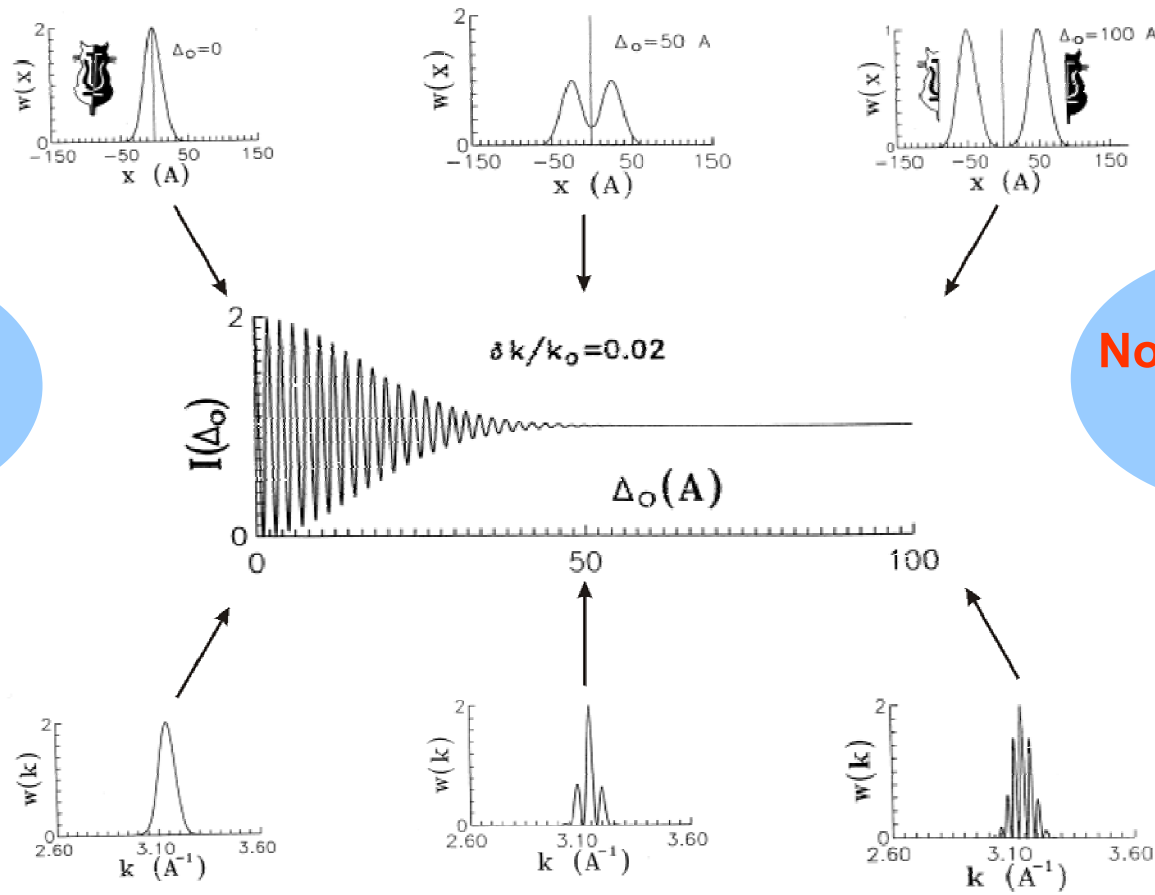
- ***Quantum State Preparation and Measurements***

Wave packet in ordinary and momentum space

Spatial distribution

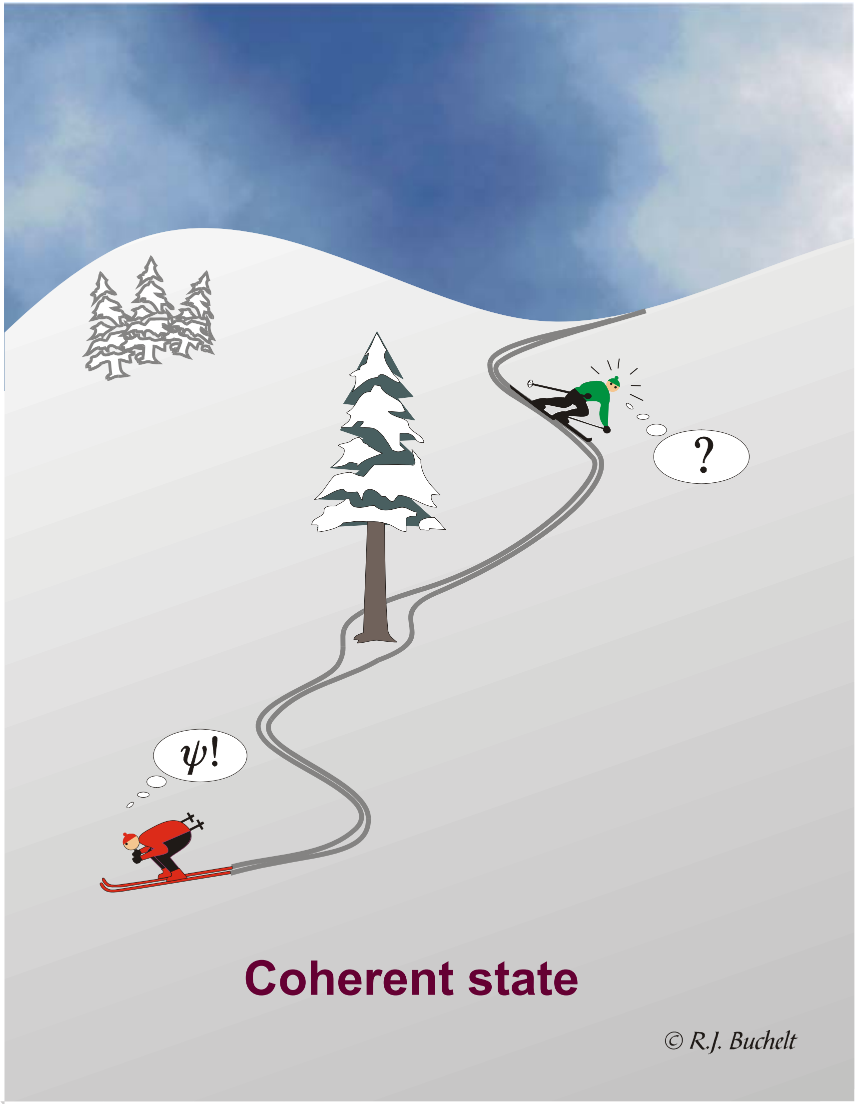
Coherent state



Non-classical state

Momentum distribution

Different quantum states



Coherent state

Non-classical state

(Schrödinger cat-like state)

State presentations

Schrödinger Equation:

$$-\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t) = i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}$$

Partial waves fill the whole space

Wave Function (Eigenvalue solution in free space):

$$\Psi(\vec{r},t) = (2\pi)^{-3/2}\int\psi(\vec{k},t)e^{i(\vec{k}\cdot\vec{r}-\omega t)}d^3\vec{k}$$

and others (Wigner function etc.)

Spatial distribution:

$$\rho(\vec{r},t) = |\psi(\vec{r},t)|^2$$

Momentum distribution:

$$g(\vec{k},t) = |\psi(\vec{k},t)|^2$$

Coherence Function:

Stationary situation: $(\tau = 0)$:

$$\Gamma(\vec{\Delta}) = <\psi(0)\,\psi(\vec{\Delta})> = (2\pi)^{3/2}\int g(\vec{k})e^{i\vec{k}\vec{r}}d^3k$$

$$\tau = t - t'$$

$$\vec{\Delta} = \vec{r} - \vec{r}\,';$$

WIGNER FUNCTION

Definition:

$$W(k,x) = \frac{1}{4\pi}\int e^{ik\Delta}\psi^*\left(x+\frac{\Delta}{2}\right)\psi\left(x-\frac{\Delta}{2}\right)d\Delta$$

Properties:

$$\int W(k,x)dk = |\psi(x)|^2$$

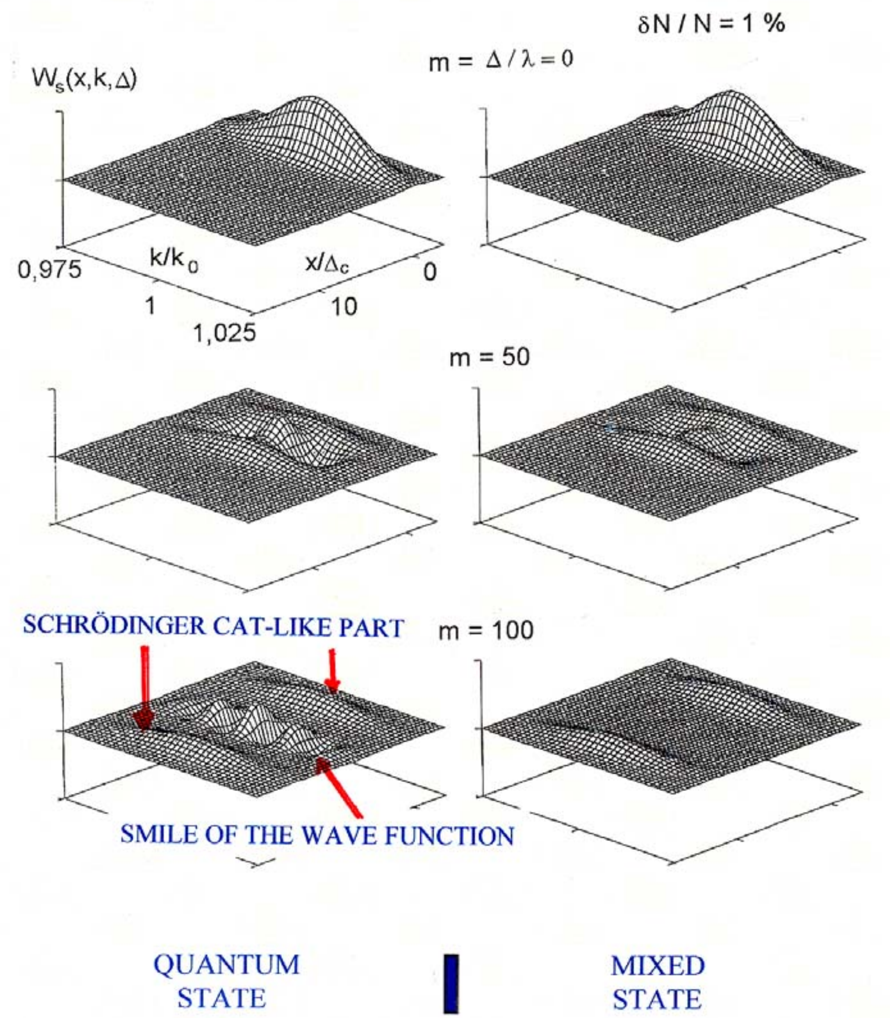
$$\int W(k,x)dx = |\psi(k)|^2$$

Interferometric Gaussian packets:

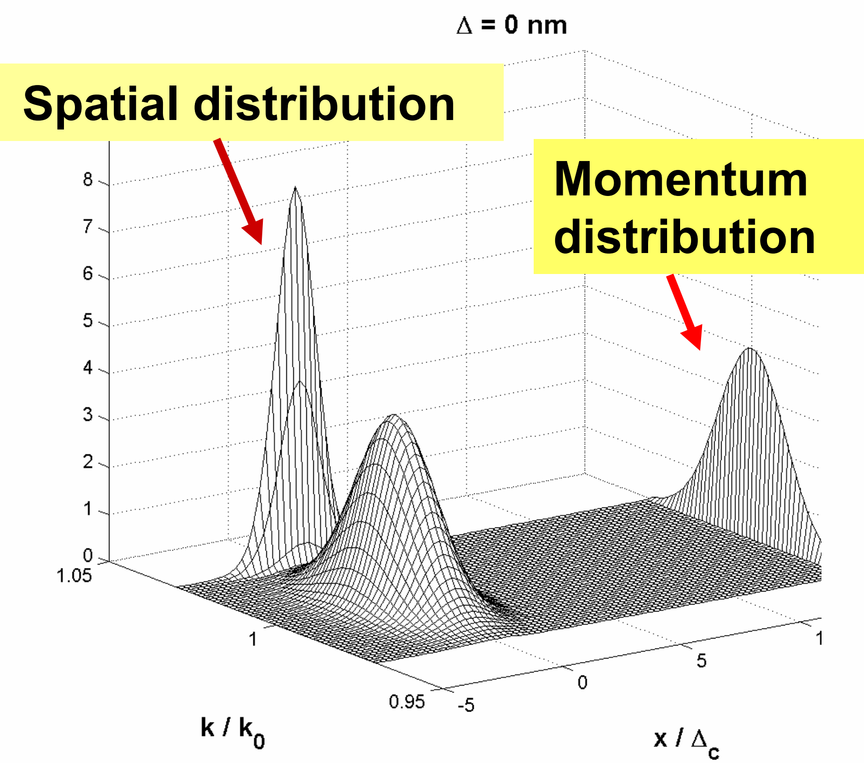
$$\psi^{I,II}(x) = \left(4\pi\delta x^2\right)^{-1/4}\exp\left[-x^2/2\delta x^2 + ixk_0\right]$$

$$\psi(x) = \psi^I(x) + \psi^{II}(x+\Delta_0)$$

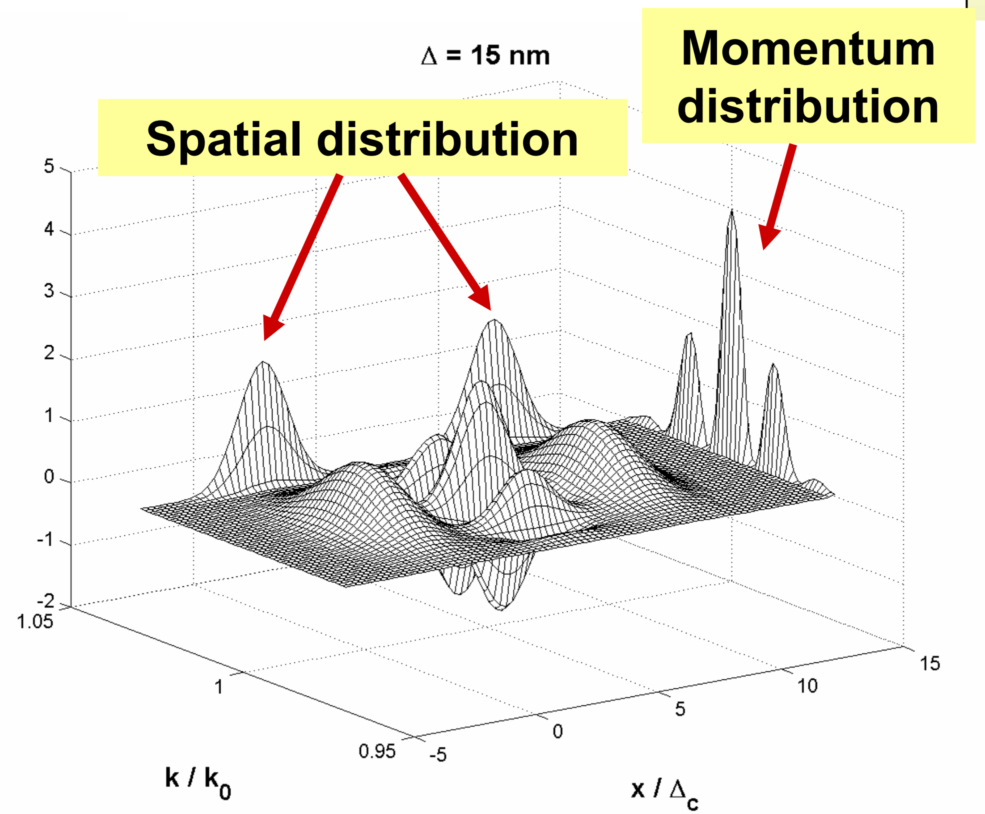
H. Rauch, M. Suda, Appl.Phys.B60 (1994) 181



WIGNER FUNCTIONS



Zero order
(coherent state)



High order
(non-classical state)

State presentations

Schrödinger Equation:

$$-\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t) = i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}$$

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Spatial distribution: **Momentum distribution:**

$$\rho(\vec{r},t) = \left|\psi(\vec{r},t)\right|^2 \qquad g(\vec{k},t) = \left|\psi(\vec{k},t)\right|^2$$

Coherence Function: $\vec{\Delta} = \vec{r} - \vec{r}\,'; \quad \tau = t - t'$

Stationary situation: $(\tau = 0)$:

$$\Gamma(\vec{\Delta}) = <\psi(0)\,\psi(\vec{\Delta})> = (2\pi)^{3/2}\int g(\vec{k})e^{i\vec{k}\vec{r}}d^3k$$

Wigner Function:

$$W(x,k) = (2\pi)^{-1}\int e^{ikx'}\psi^*\left(x+\frac{x'}{2}\right)\psi\left(x-\frac{x'}{2}\right)dx'$$

Q-Function (Husimi-Function):

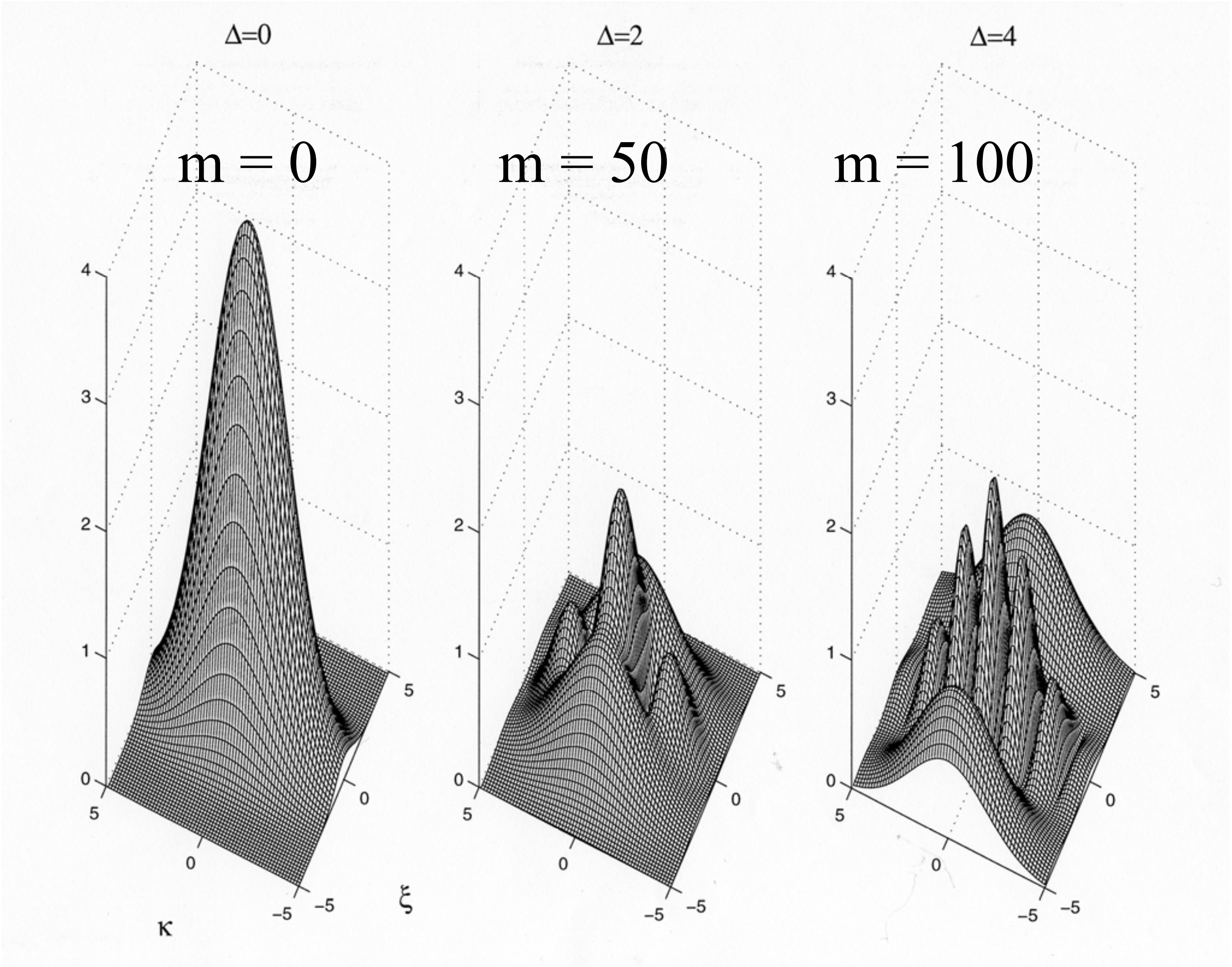
$$Q(x,k) = \iint W(x',k')\,g(x-x',k-k',\gamma)\,dx'dk'$$

$$g(x-x',k-k') \propto \exp\left[-\frac{(x-x')^2}{\gamma} - \gamma(k-k')^2\right]$$

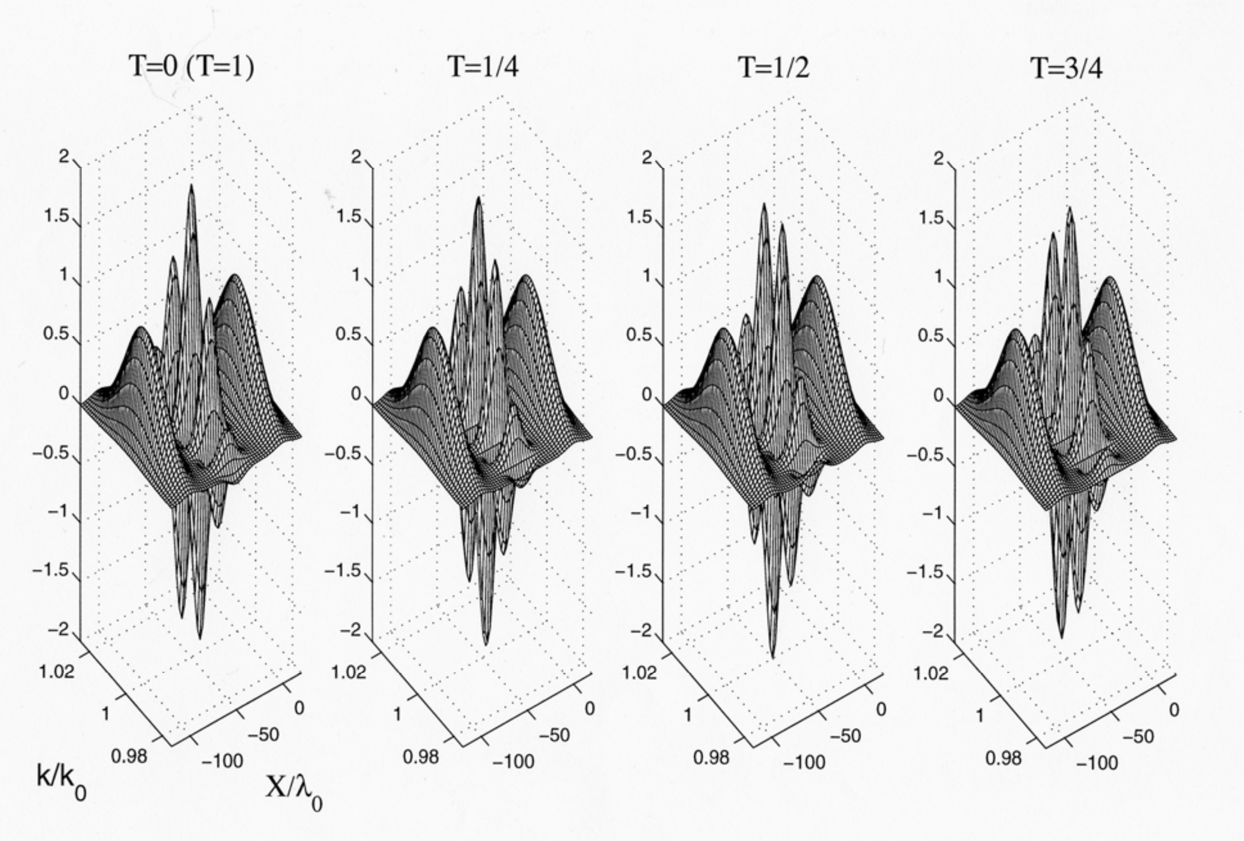
Weyl Function:

$$\tilde{W}(X,K) = \iint W(x,k)\,e^{-i(Kx-kX)}\,dkdx$$

Weyl – Function (modulus)



H.Rauch, M.Suda, Lect.Notes in Physics, Springer 2002 m = 100



H.Rauch, M.Suda, Lect.Notes in Physics, Springer 2002

Quantum State Reconstruction

via Wigner functions

$$W(x,k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx'}\psi^*(x+\frac{x'}{2})\,\psi(x-\frac{x'}{2})\,dx'$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx'} < x+\frac{x'}{2})\,|\hat{\rho}|\;\; x-\frac{x'}{2} > dx'$$

$$(x \;\hat{=}\; \Delta = -Nb_c\,\lambda^2 D/2\pi)$$

Quadrature operator

$$\hat{X}_\Theta = k_0\,\hat{x}\cos\Theta + \frac{\hat{k}}{k_0}\sin\Theta$$

Quadrature Wigner function

$$W(X_\Theta) = \frac{\hbar}{2\pi}\int_{-\infty}^{+\infty} dt\,e^{-itX_\Theta}\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dk\;\; e^{it(k_0 x\cos\Theta + k\sin\Theta/k_0)}\,W(x,k)$$

Radon transformation

$$W(x,k) = \frac{1}{4\pi^2\hbar}\int_{-\infty}^{+\infty}dt|t|\int_0^{\pi}d\Theta\int_{-\infty}^{+\infty}dX_\Theta e^{it(X_\Theta - k_0 x\cos\Theta - k\sin\Theta/k_0)}\,\mathbf{W(X_\Theta)}$$

Neutron interferometry case (Gaussian packets)

$$W(x,k,\Delta) = W(x,k) + W(x+\Delta,k) + 2\cos k\Delta W(x+\frac{\Delta}{2},k)$$

$$W(X_\Theta) = \frac{\hbar}{2\pi}\sqrt{\frac{\pi}{b}}\Big\{e^{-(X_\Theta-\sin\Theta)^2/4b}$$

$$+ \quad e^{-(X_\Theta-\sin\Theta+k_0\Delta\cos\Theta)^2/4b}$$

$$+ \quad 2e^{-(X_\Theta-\sin\Theta+k_0\Delta\cos\Theta)^2/4b}$$

$$\cdot \quad e^{-\sigma^2(k_0\Delta)^2/2+\sigma^4(k_0\Delta)^2\sin^2\Theta/4b}$$

$$\cdot \quad \cos[(k_0\Delta)+(X_\Theta-\sin\Theta+k_0\Delta\cos\Theta/2)\,\sigma^2(k_0\Delta)\sin\Theta)/2b]\Big\}$$

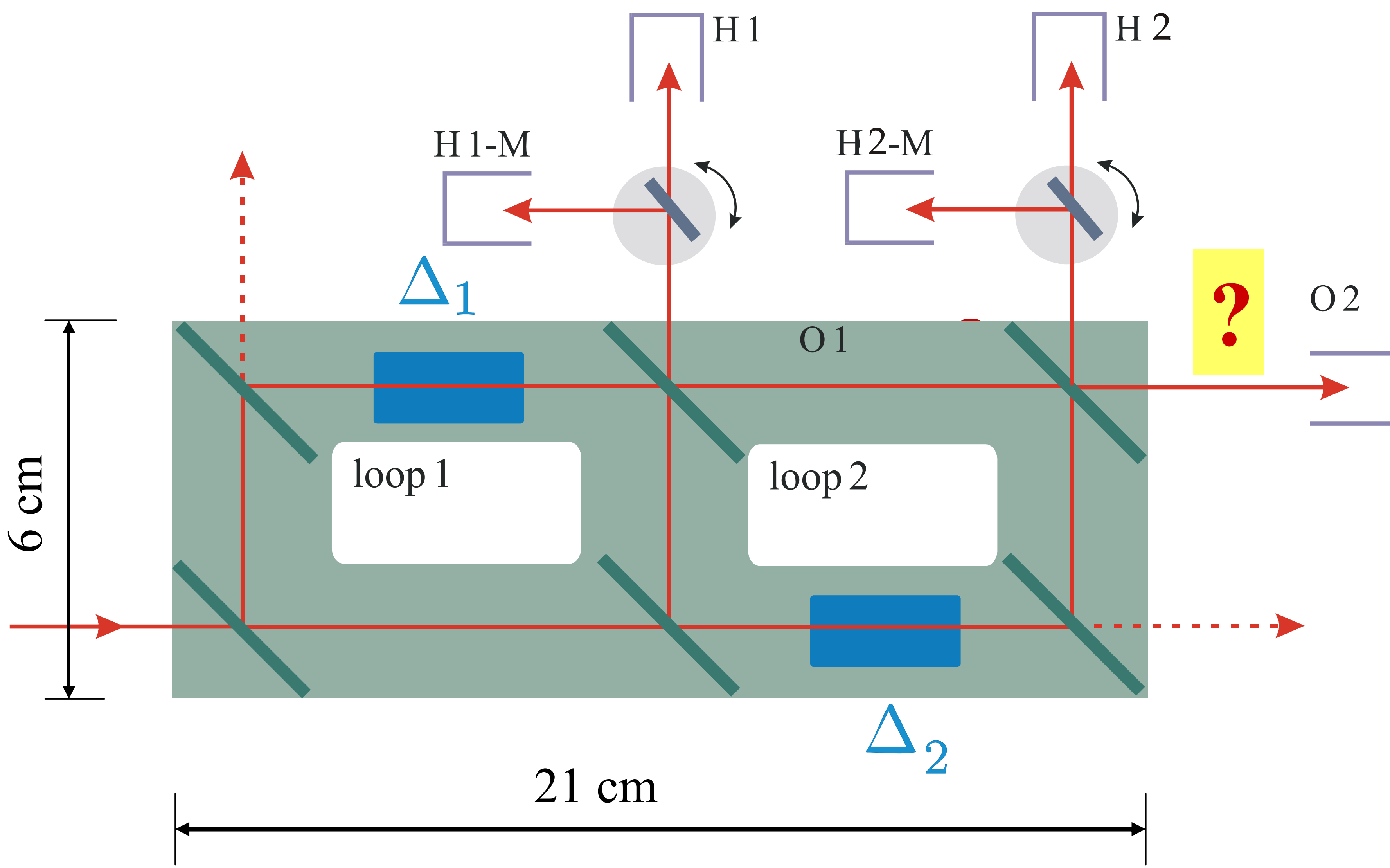
with:

$$b = \cos^2\Theta/8\sigma^2 + \sigma^2\sin^2\Theta/2$$

$$\sigma = \delta k/k_0$$

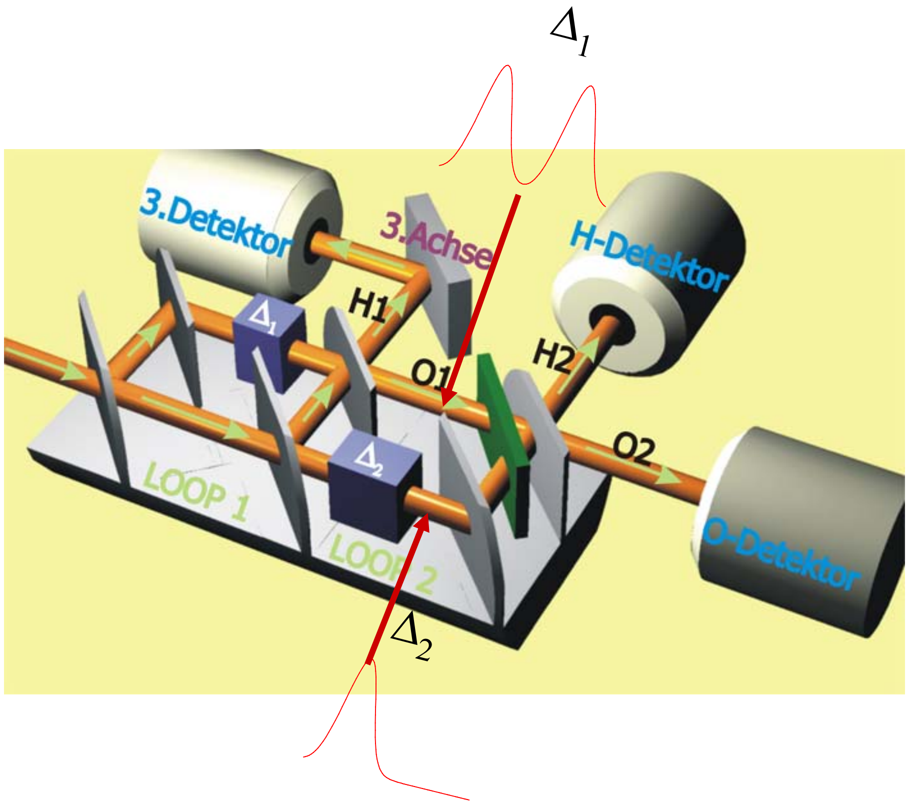
H. Rauch and M. Suda, Appl.Phys.A2002

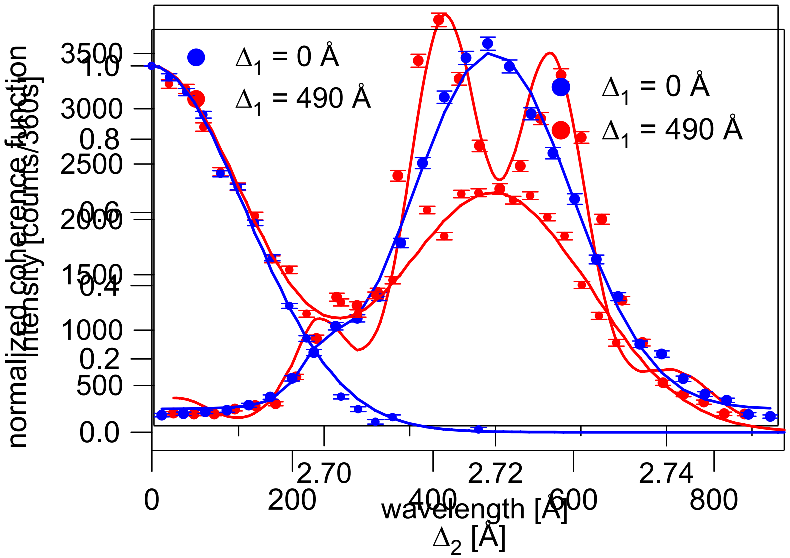
TWO LOOP INTERFEROMETER



M.Zawisky, M.Baron, R.Loidl, H.Rauch, Nucl.Instr.Meth. A481 (2002) 406

Wave Packet Structure

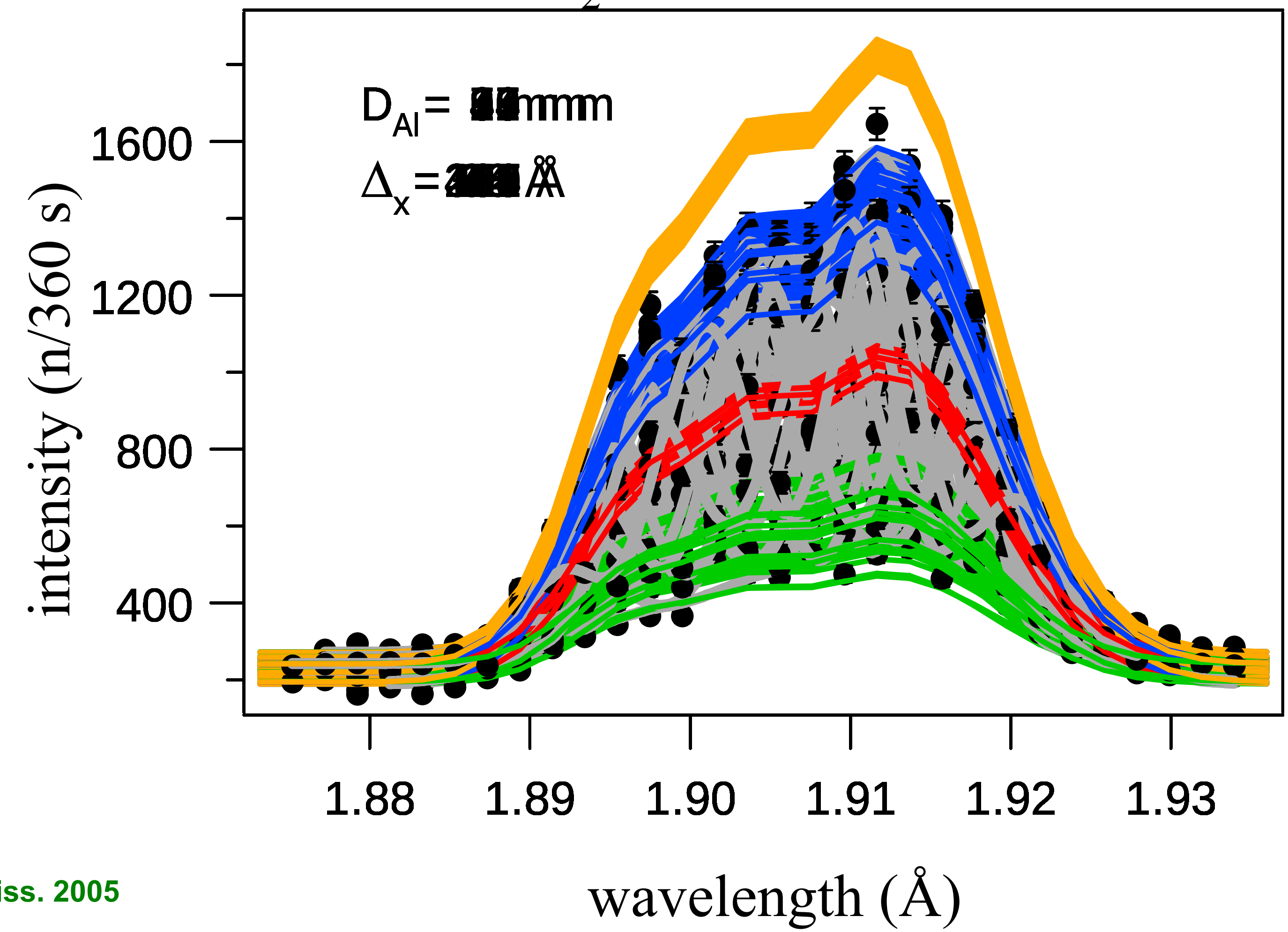




M.Baron, H.Rauch, M.Suda, J.Opt.B5 (2003) S341

Momentum Modulation

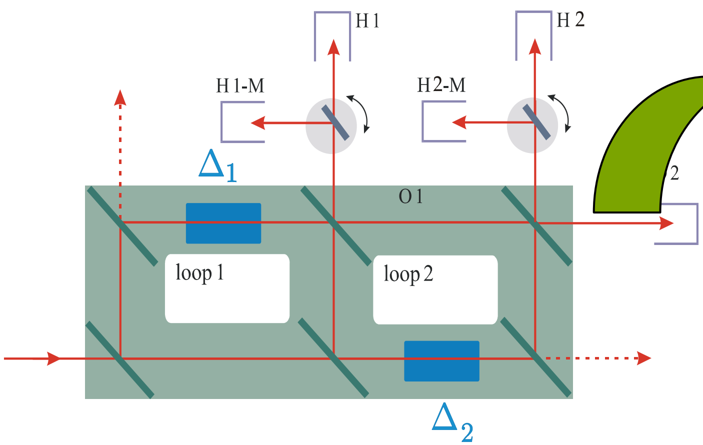
$$I_0(\vec{k}) = \frac{1}{2} g(\vec{k}) \cdot \left(1 + C \cos \vec{\Delta}\vec{k}\right)$$

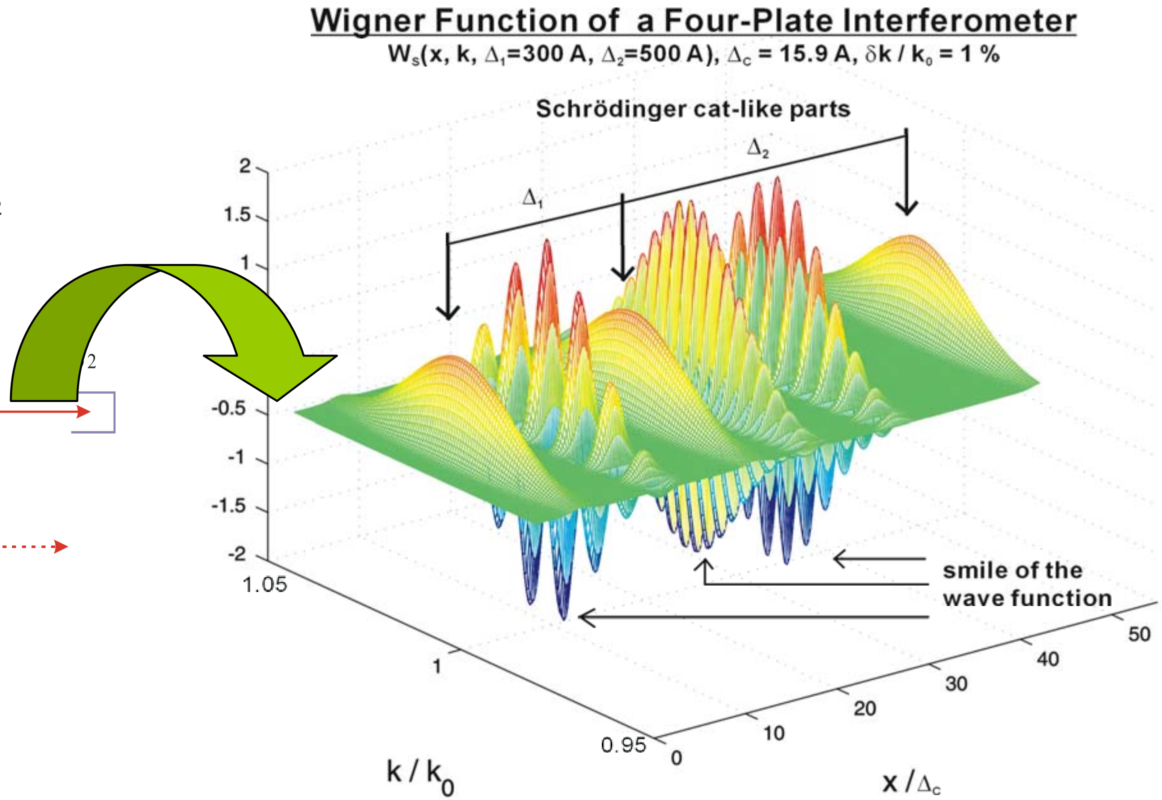


M.Baron, Diss. 2005

Triple Wigner Function

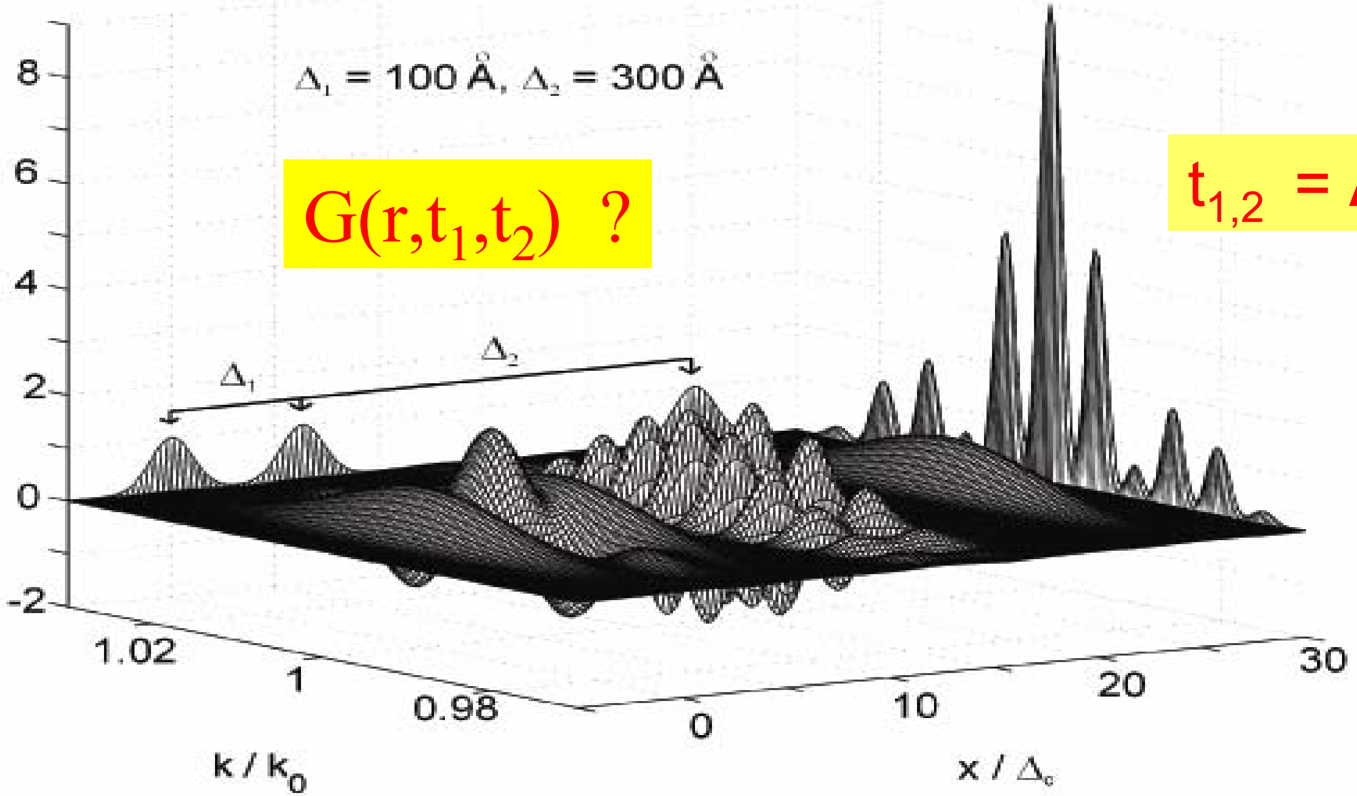
TWO LOOP INTERFEROMETER





Quantum state engineering !!!

Triple Peak Wigner Function



$G(r,t_1,t_2)$?

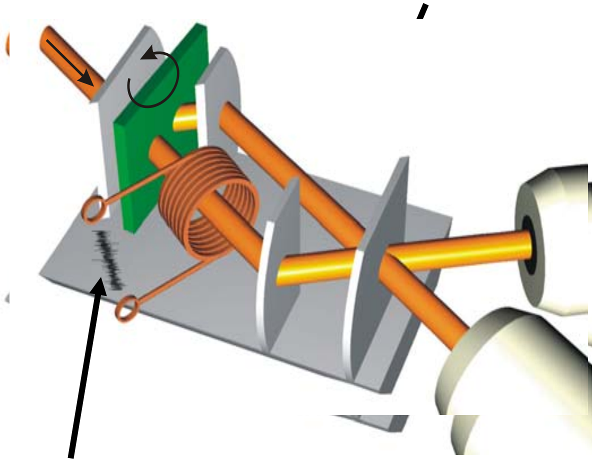
$t_{1,2} = \Delta_{1,2}/v$

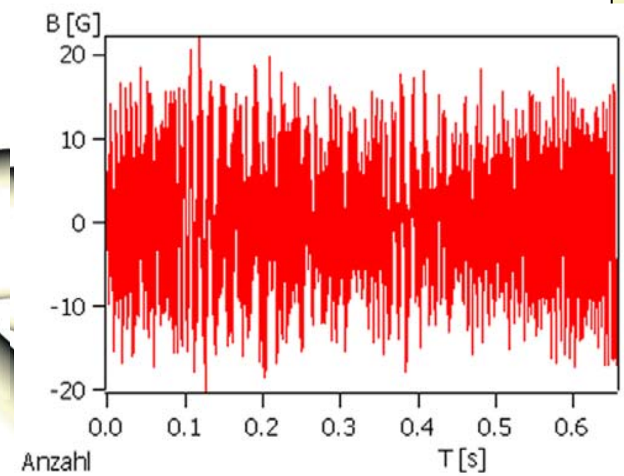
H.Rauch, M.Suda 2001

- ***Magnetic Noise Dephasing or Decoherencing***

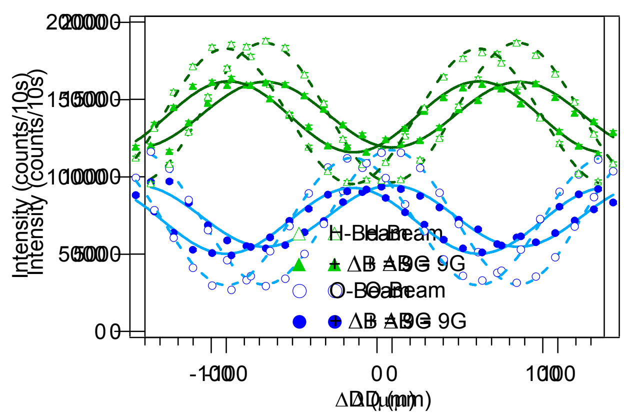
Dephasing at low order

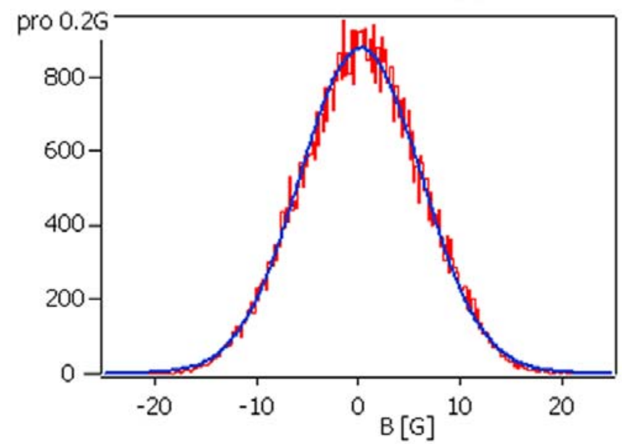
Magnetic noise field





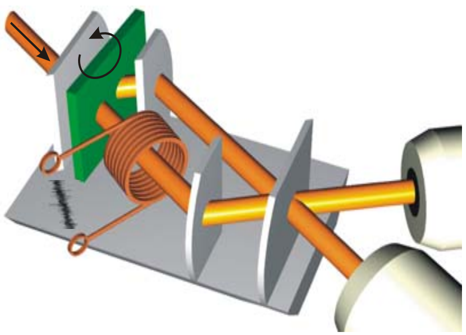
Magnetic noise field

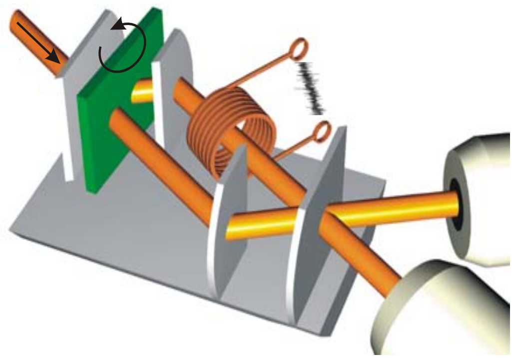


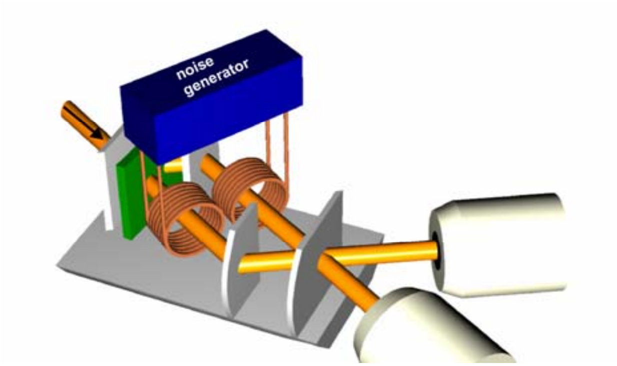


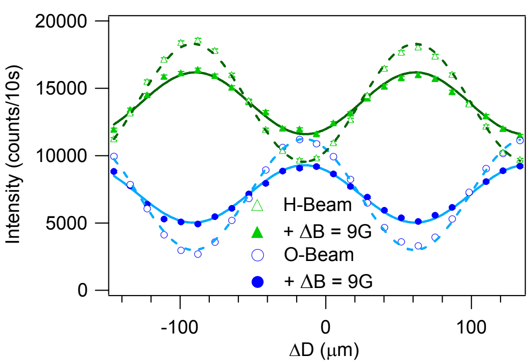
M.Baron, H.Rauch, M.Suda (in progress)

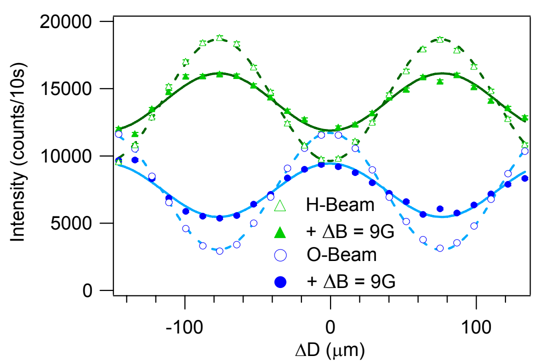
Magnetic noise fields

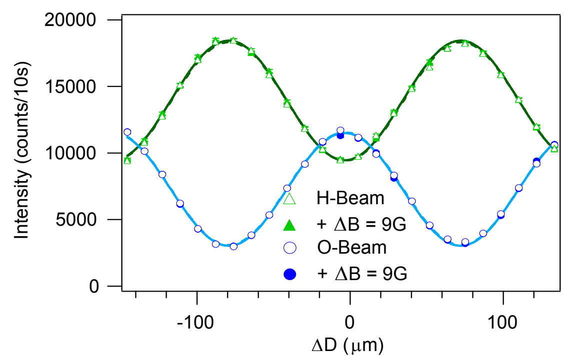






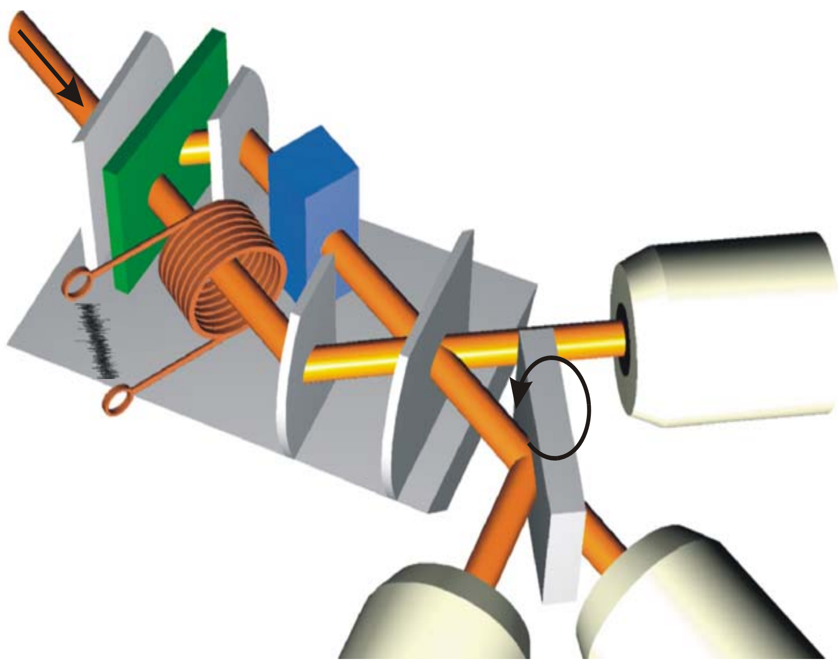


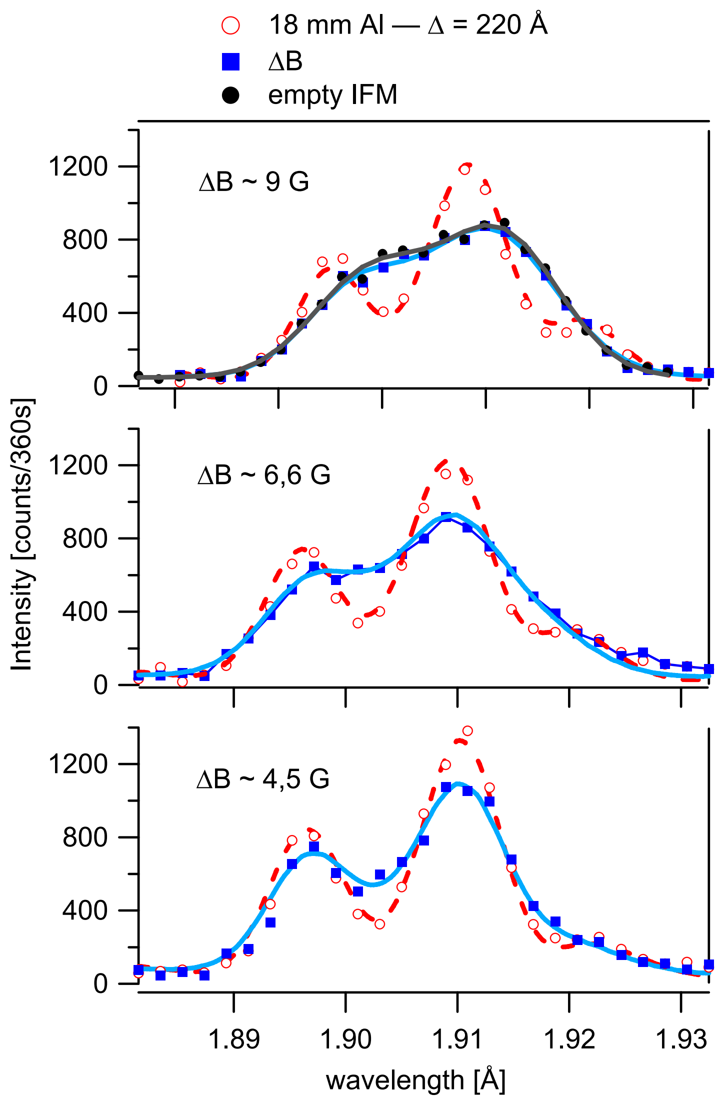




M.Baron, H.Rauch, M.Suda (in progress)

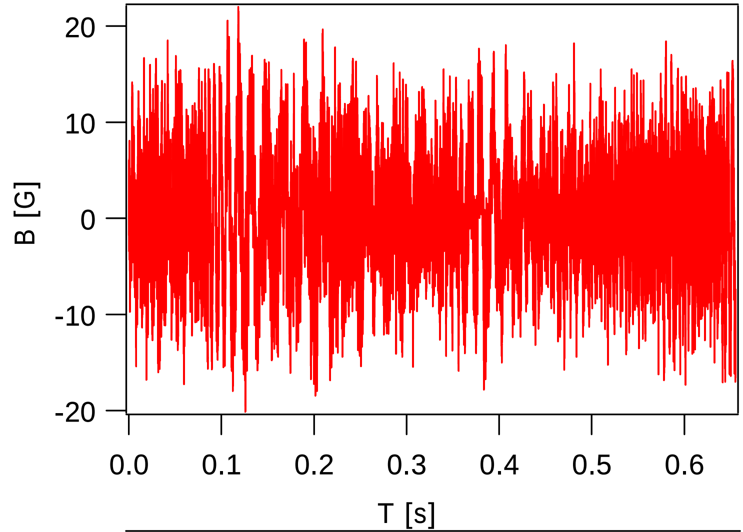
Dephasing at high order

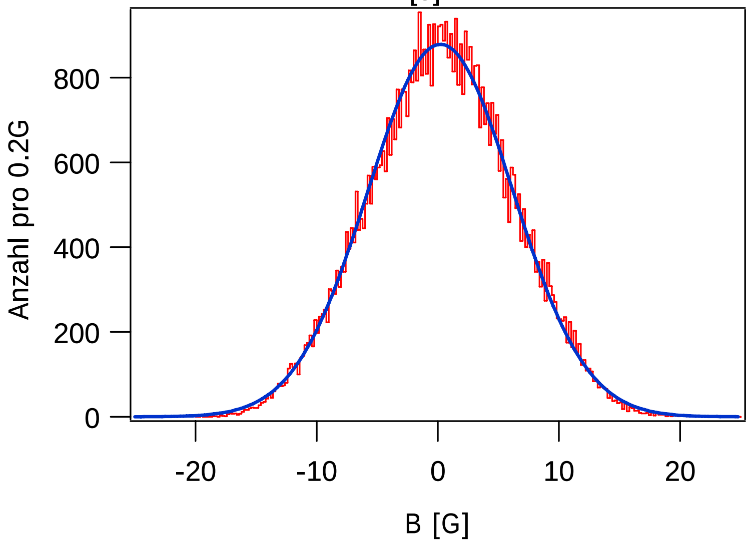


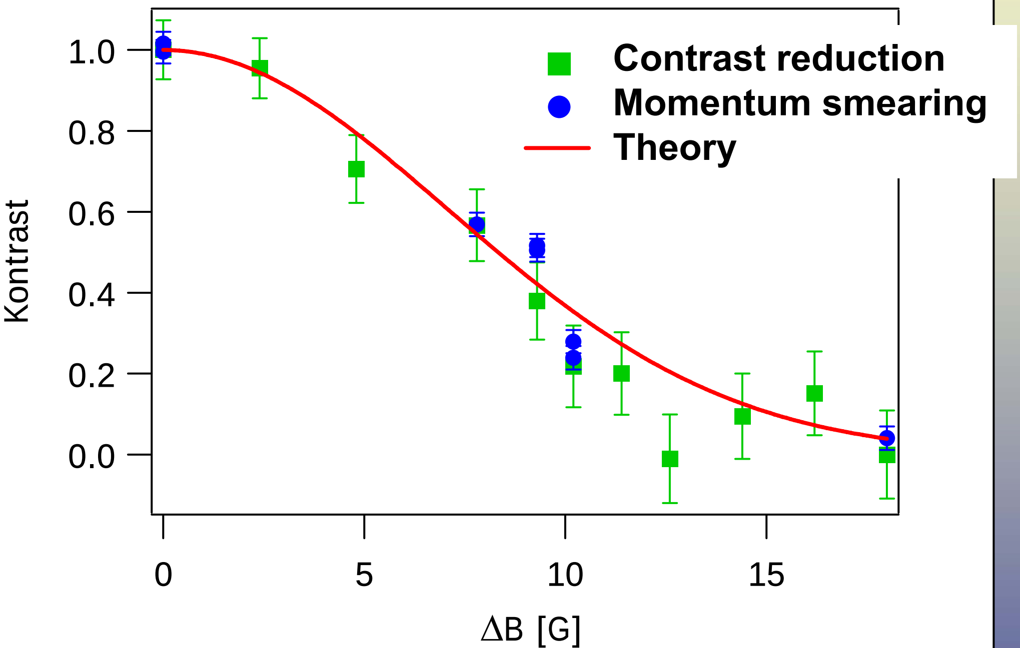


M.Baron, H.Rauch, M.Suda, J.Opt.B5 (2003) S244

Magnetic Noise Field



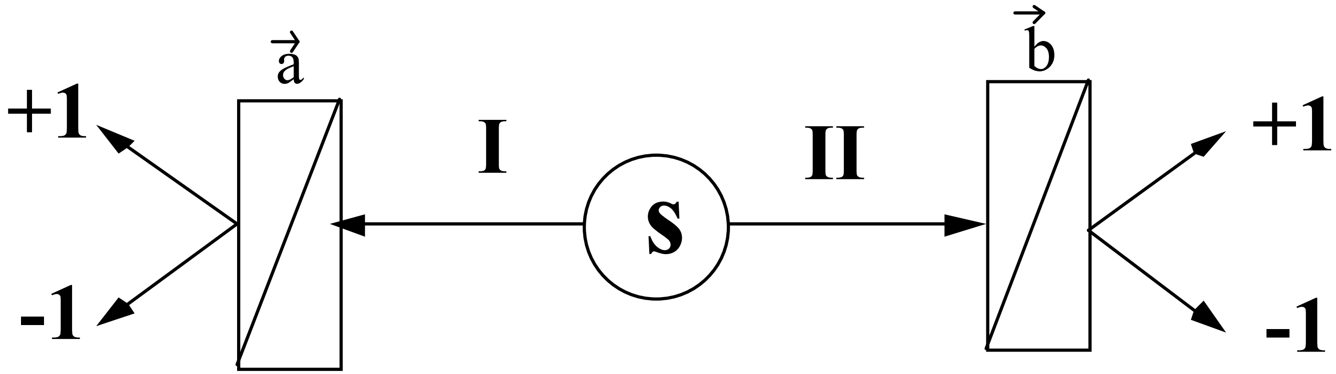




$$C = C_0 \exp[-(\mu \Delta B D_{eff}/\hbar v)^2/2]$$

- ***Quantum Contextuality***

EPR - Experiment



Entanglement of two photon polarizations

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

⇒⇒⇒ Entanglement between *Two-Particles*

A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47 (1935) 777.

Two-particle vs. two-space entanglement

2-Particle Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{I}\otimes|\downarrow\rangle_{II} + |\downarrow\rangle_{I}\otimes|\uparrow\rangle_{II}\}$$

I, II represent 2-Particles

Measurement on each particle

$\hat{A}^{I}($ …

$\hat{B}^{II}($ …

where $\hat{P}_{(\vec{\xi};\pm 1)} = \frac{1}{2}(|\uparrow\rangle \pm e^{i\xi}|\downarrow\rangle)(\langle\uparrow| \pm e^{-i\xi}\langle\downarrow|)$

Then, $[\hat{A}^{I}, \hat{B}^{II}] = 0$

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{s}\otimes|\,I\rangle_{p} + |\downarrow\rangle_{s}\otimes|\,II\rangle_{p}\}$$

s, p represent 2-Spaces, e.g., spin

Measurement on each property

… π)

… π)

where $\hat{P}_{(\phi)} = \frac{1}{2}(|\varphi\rangle + e^{i\phi}|\bar{\varphi}\rangle)(\langle\varphi| + e^{-i\phi}\langle\bar{\varphi}|)$

Then, $[\hat{A}^{s}, \hat{B}^{p}] = 0$

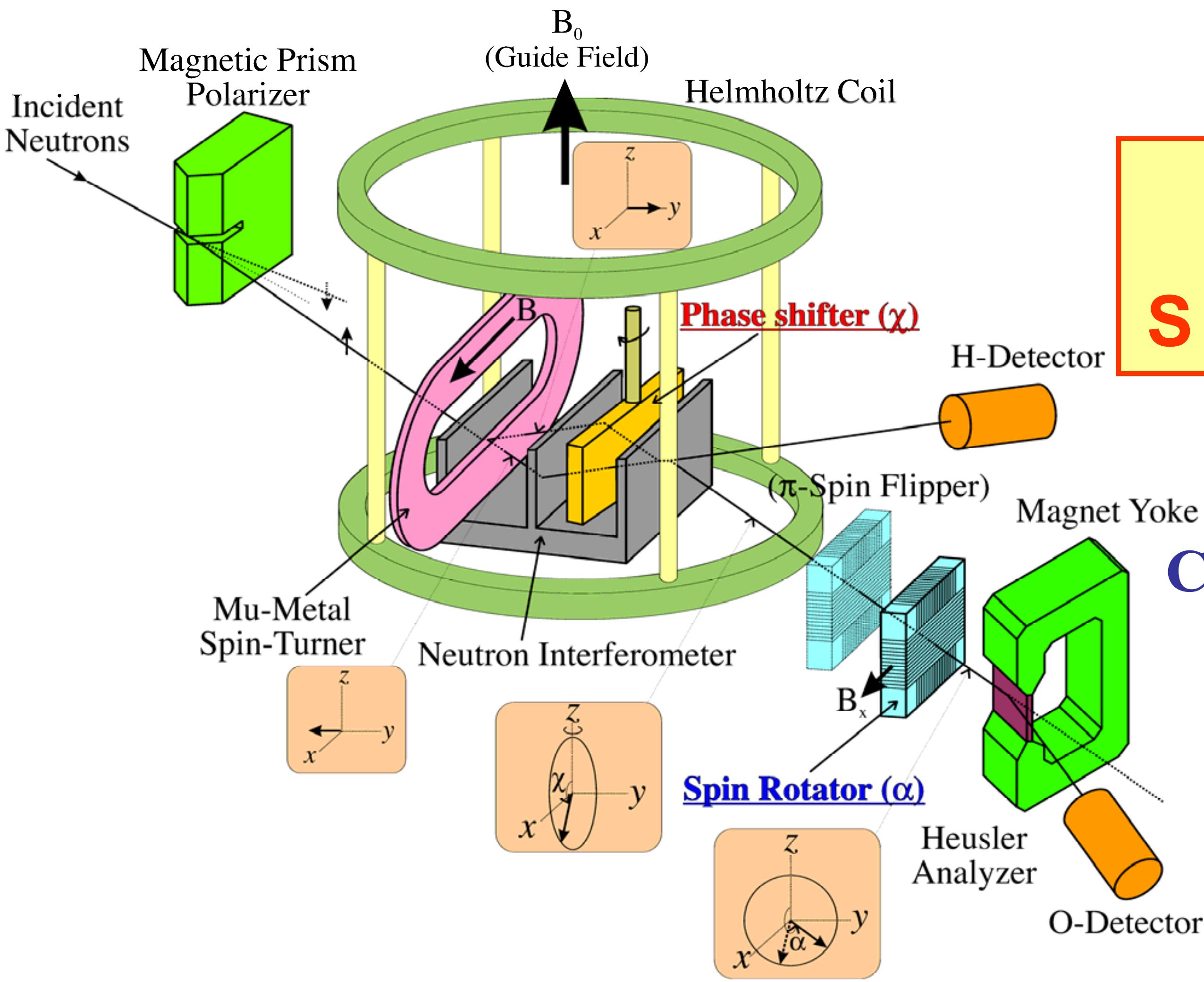
$-2 < S < 2$

$S = E(\alpha_1,\chi_1) - E(\alpha_1,\chi_2) + E(\alpha_2,\chi_1) + E(\alpha_2,\chi_2)$

==>> **(Non-)Contextuality**

(In)Dependent Results for commuting Observables

Contextuality Experiment



Result:

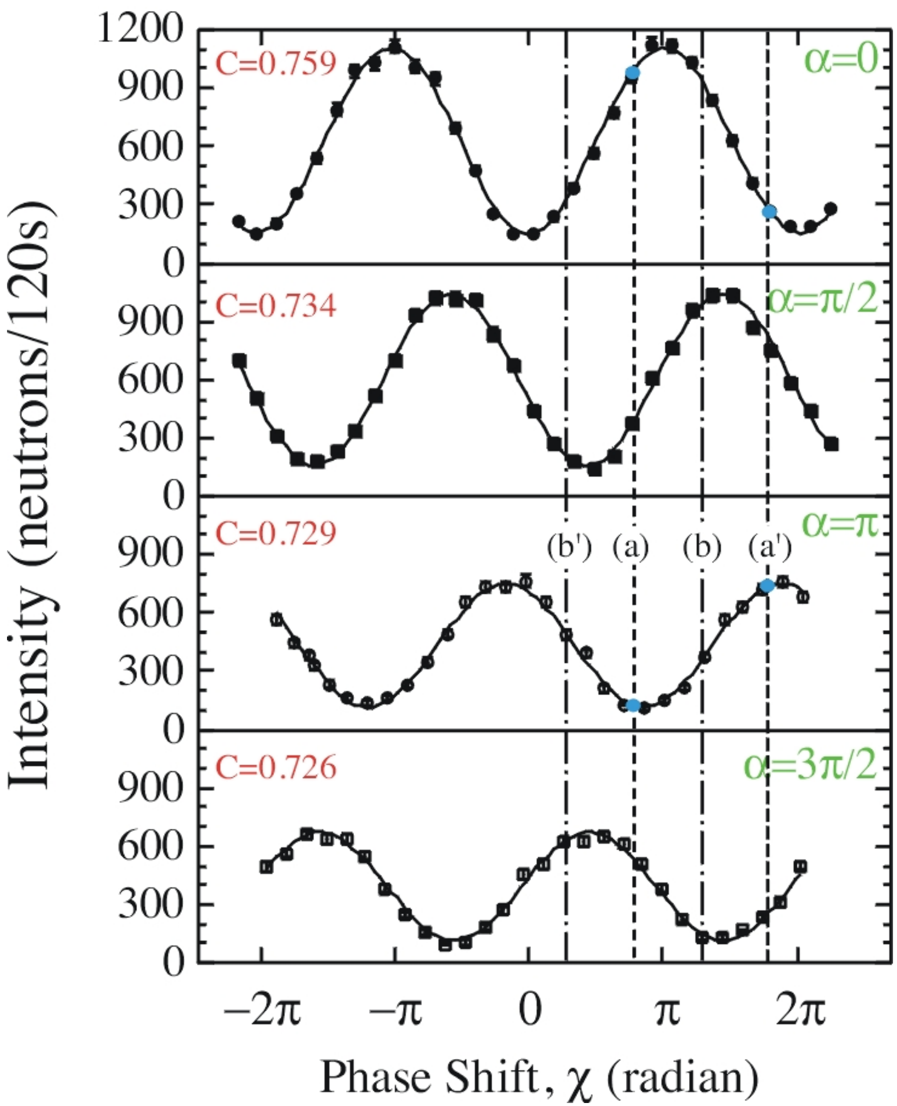
S = 2.051 ± 0.019

Theory: S_{Max} = 2.82

Classical correlation:
(hidden variables)
$-2 < S' < 2$

.Y.Hasegawa, R.Loidl, G.Badurek, M.Baron, H.Rauch, Nature 425 (2003) 45 and Phys.Rev.Lett.97 (2006) 230401

Contextuality Results

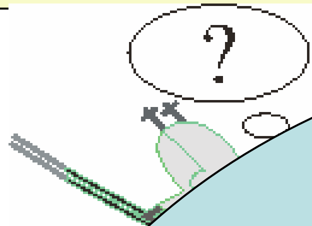


$$E'(\alpha=0,\ \chi = 0.79\pi)$$
$$= [N'(0,0.79\pi) + N'(\pi,1.79\pi) - N'(0,1.79\pi) - N'(\pi,0.79\pi)]$$
$$\div [N'(0,0.79\pi) + N'(\pi,1.79\pi) + N'(0,1.79\pi) + N'(\pi,0.79\pi)]$$
$$= 0.542$$

In the same manner,

$$\begin{cases} E'(\alpha=0,\ \chi = 1.29\pi) \\ E'(\alpha=0.5\pi,\ \chi = 0.79\pi) \\ E'(\alpha=0.5\pi,\ \chi = 1.29\pi) \end{cases}$$

were determined.



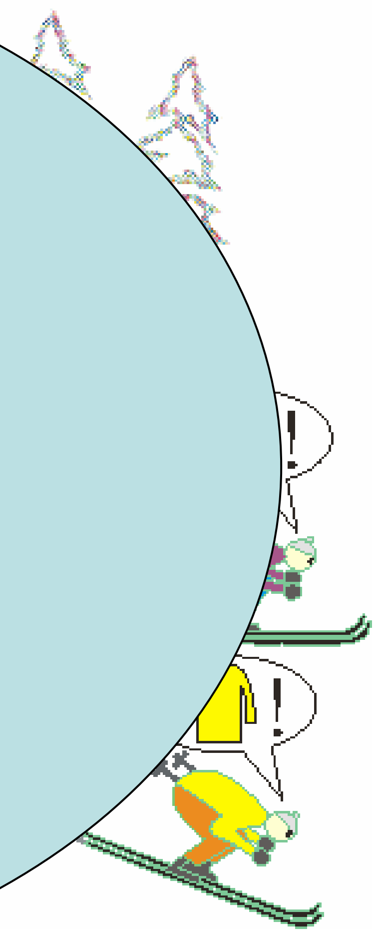
Kochen-Specker phenomenon

$$C_{non\text{-}contextual} = C_{classic} = 2$$

$$C_{contextual} = C_{quantum} = 4$$

$$C_{experimental} = 3.138(15)$$

Y.Hasegawa, R.Loidl, G. Badurek, M.Baron, H.Rauch,
PRL 97 (2006) 23040



A cartoon-like

The colour of a sk
is undetermined, re
After a ‘measurement
on the path in our experi
its own colours (the direct
depending on what was
Basically no correlation … colours of jacket and trousers
is expected!

ATOMINSTITUT

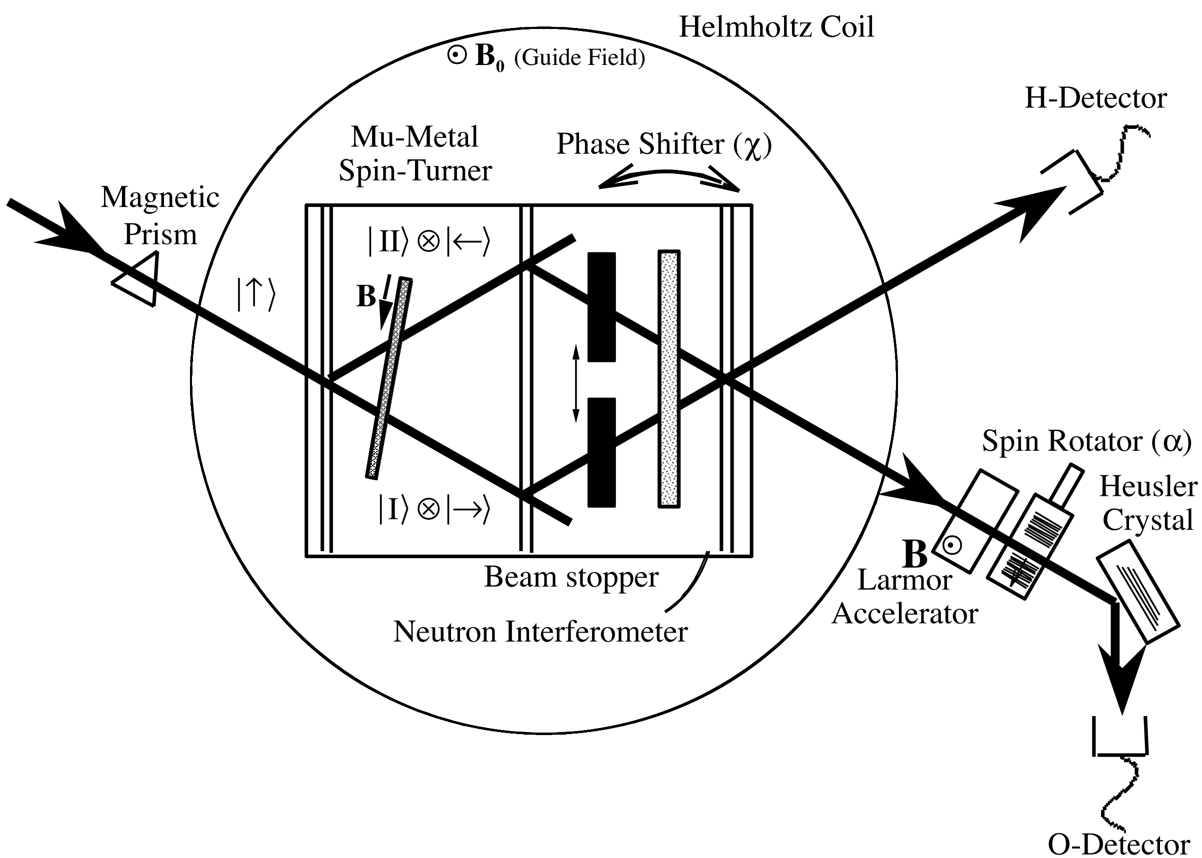
TU VIENNA

- ***Spin State Reconstruction***

Spin State Tomography

Experimental setup

Measurement principle:D.F.V.James, P.G.Kwiat, W.J.Munro, A.G.White, PR/A,64 (2001)o52312



Beam I

Beam II

Beam I+II, χ = 0, π/2

|z>,|-z>,|x>,|y>

→ 16 positions

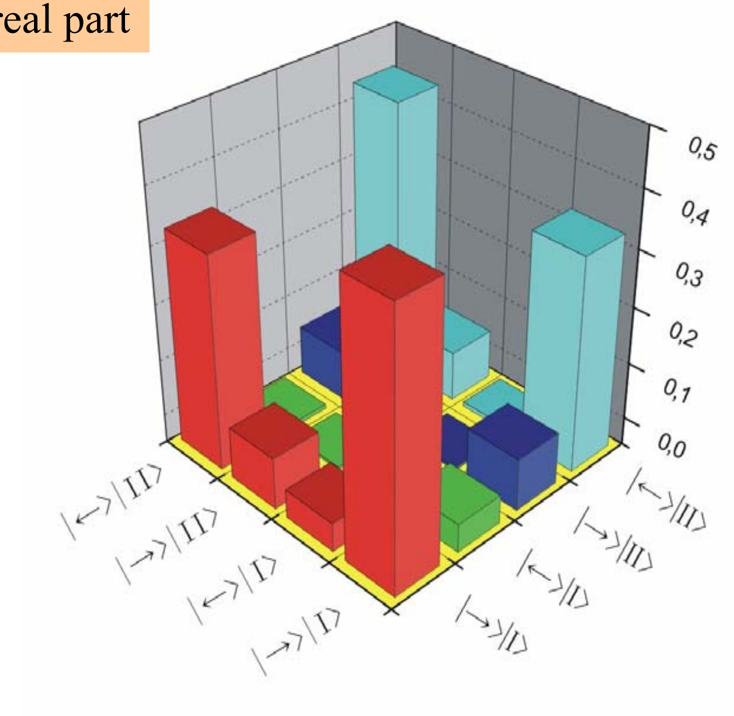
Y.Hasegawa, J.Klepp, S.Filipp, R.Loidl (in progress)

Quantum state tomography of neutron's Bell-state

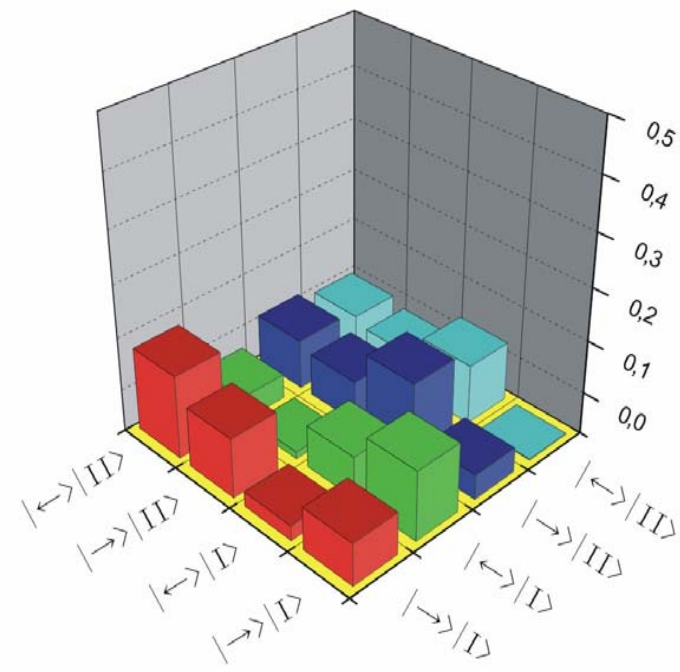
$$\rho = |\psi\rangle\langle\psi|$$

where $\langle\psi| = \{\langle\uparrow|, \langle\downarrow|\} \otimes \{\langle I|, \langle II|\}$

real part



imaginary part



Y.Hasegawa, J.Klepp, S.Filipp, R.Loidl (in progress)

- ***Geometrical Phases***

Berry Phase (adiabatic & cyclic evolution)

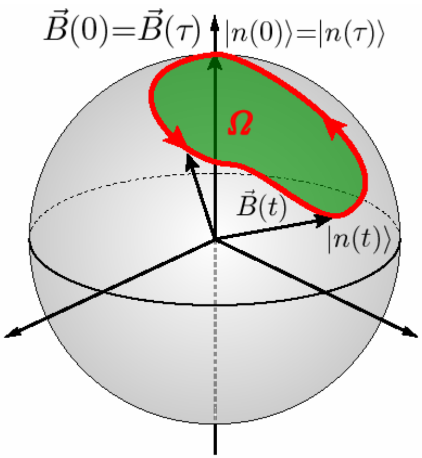
[Berry; Proc.R.S.Lond. A 392, 45 (1984)]

$$|\psi(t)\rangle = e^{-i\phi_d} e^{i\phi_g} |n(R(t))\rangle$$

$$\phi_d(t) = \frac{1}{\hbar}\int_0^t dt' E_n(t')$$

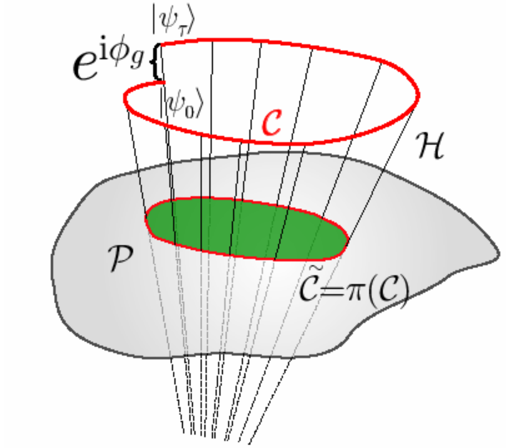
$$\phi_g = -\frac{\Omega}{2}$$

(for 2-level systems)



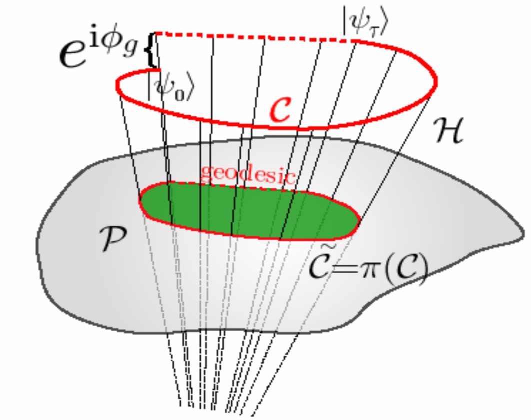
Non-adiabatic evolution

[Aharonov & Anandan, PRL 58, 1593 (1987)]



Non-adiabatic & non-cyclic evolution

[Samuel & Bhandari, PRL 60, 2339 (1988)]



Topological Phase

Theory:

S.Pancharatnam, Proc.Ind.Acad.Sci. A44(1956)247
M.V.Berry, Proc.Roy.Soc.London, A392(1984)415
J.Anandan,Nature360(1992)307; R.Bhandari,Phys.Rep.281(1977)1

$$|\psi(T)> \;=\; e^{i\phi}\,|\psi(0)>$$

$$\Downarrow$$

$$\phi \;=\; -\frac{1}{\hbar}\int_0^T <\psi(t)|H|\psi(t)> dt + i\int_0^T <\psi(t)|\frac{d}{dt}|\psi(t)> dt = \delta + \gamma$$

$$\delta \;=\; \frac{\alpha}{2}\cos\Theta \quad \text{.......... dynamical phase}$$

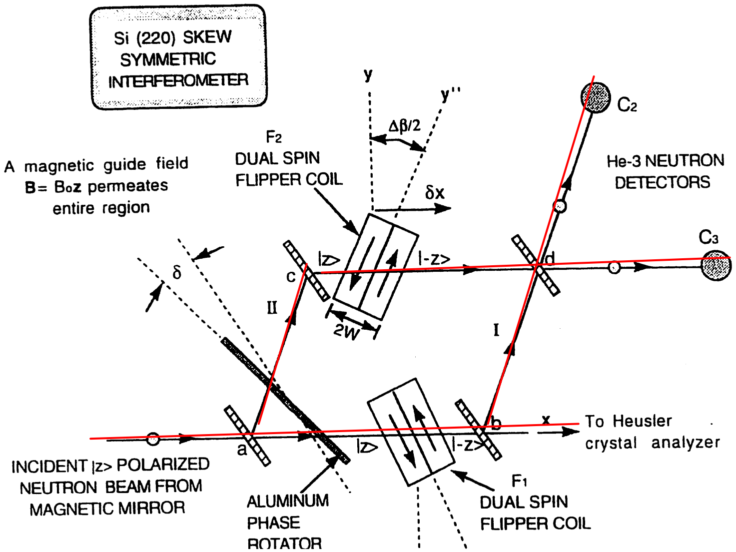
$$\gamma \;=\; \frac{\alpha}{2}(1-\cos\Theta) \quad \text{... geometric phase}$$

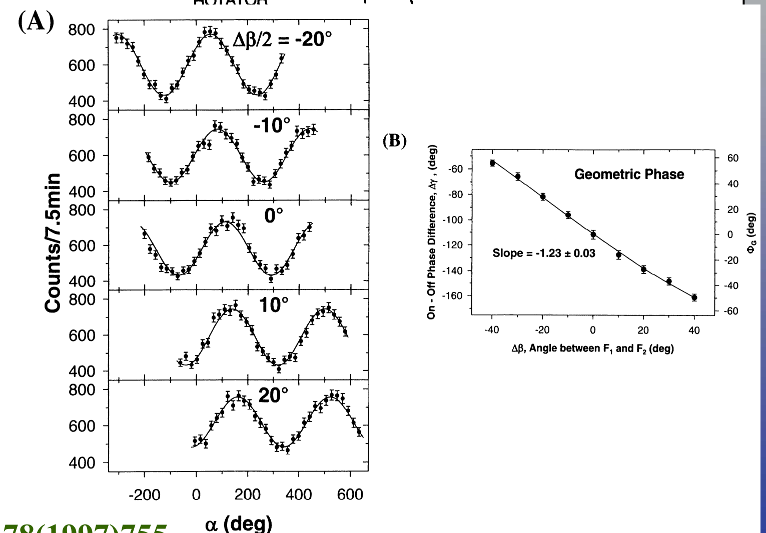
$$\Downarrow$$

$$I(\chi,\alpha) \;=\; |\psi_0(0,0)+\psi_0(\chi,\alpha)|^2 \propto D + \cos\chi\cos\frac{\alpha}{2} + \sin\chi\sin\frac{\alpha}{2}\cos\Theta$$

$$= \;D + A\cos(\chi+\phi)$$

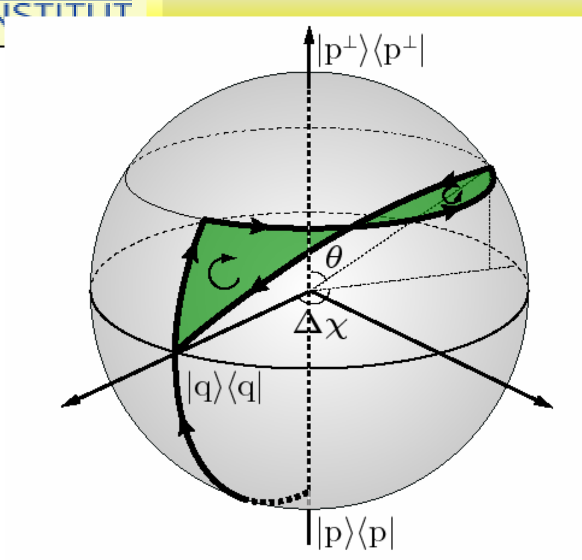
$$A \;=\; \sqrt{1-\sin^2\Theta\sin^2\frac{\alpha}{2}} \qquad \cos\phi = \frac{\cos\frac{\alpha}{2}}{\sqrt{\cos^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}\cos^2\Theta}}$$

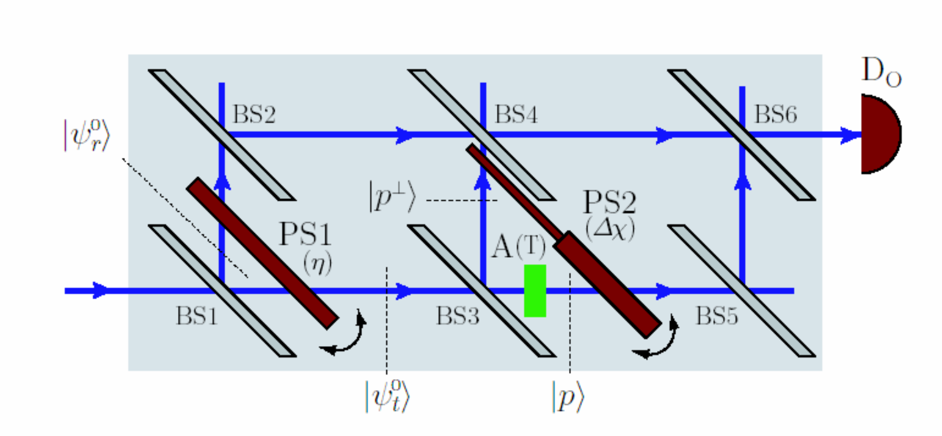




Exp.: A.G.Wagh, V.C.Rakhecha, J.Summhammer, G.Badurek, H.Weinfurter, B.E.Allman, H.Kaiser, K.Hamacher, D.L.Jacobson, S.A.Werner, Phys.Rev.Lett. 78(1997)755

Non-adiabatic & Non-cyclic Phase





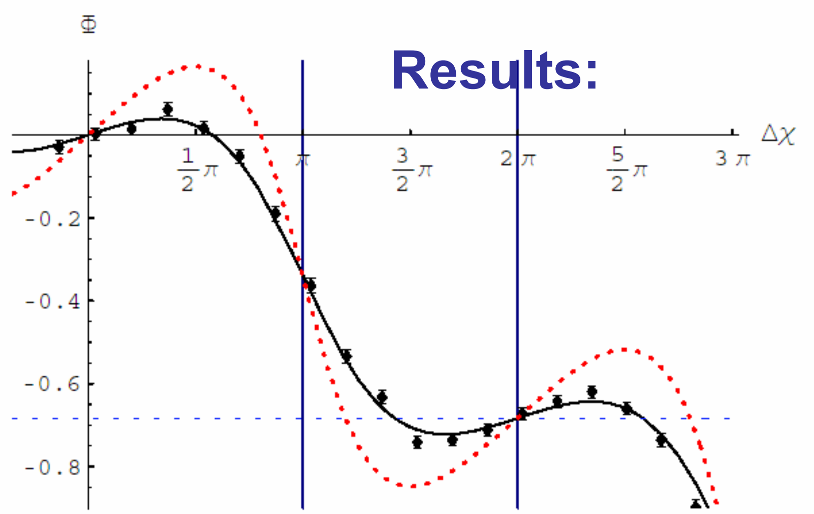
$$\Phi \equiv \arg\langle\psi'_r|\psi'_t\rangle = \frac{\chi_1+\chi_2}{2} - \arctan\left[\tan\frac{\Delta\chi}{2}\left(\frac{1-\sqrt{T}}{1+\sqrt{T}}\right)\right]$$

$$\Phi_g \equiv \Phi - \Phi_d = \Phi$$

Cancelling dynamical phase, if

$$\Phi_d = \frac{\chi_1 + T\chi_2}{1+T} = 0$$

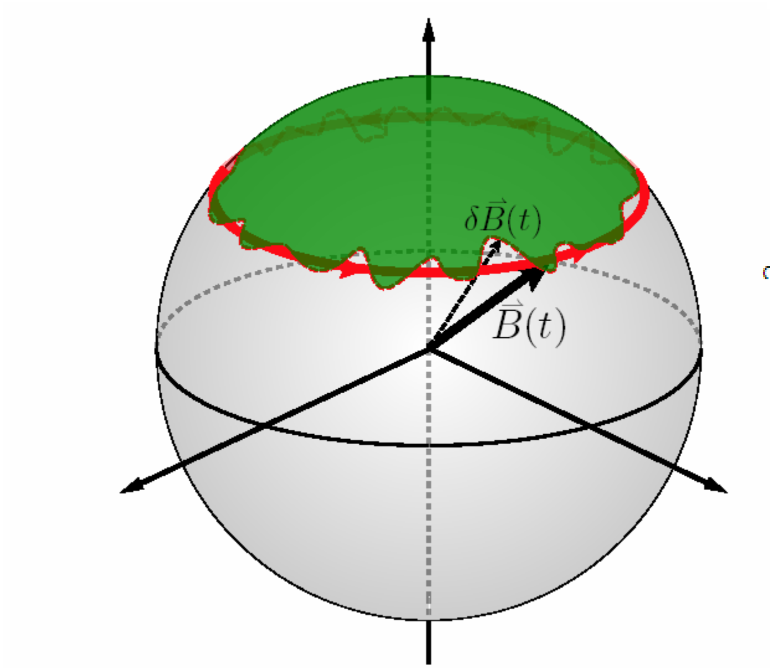
Results:

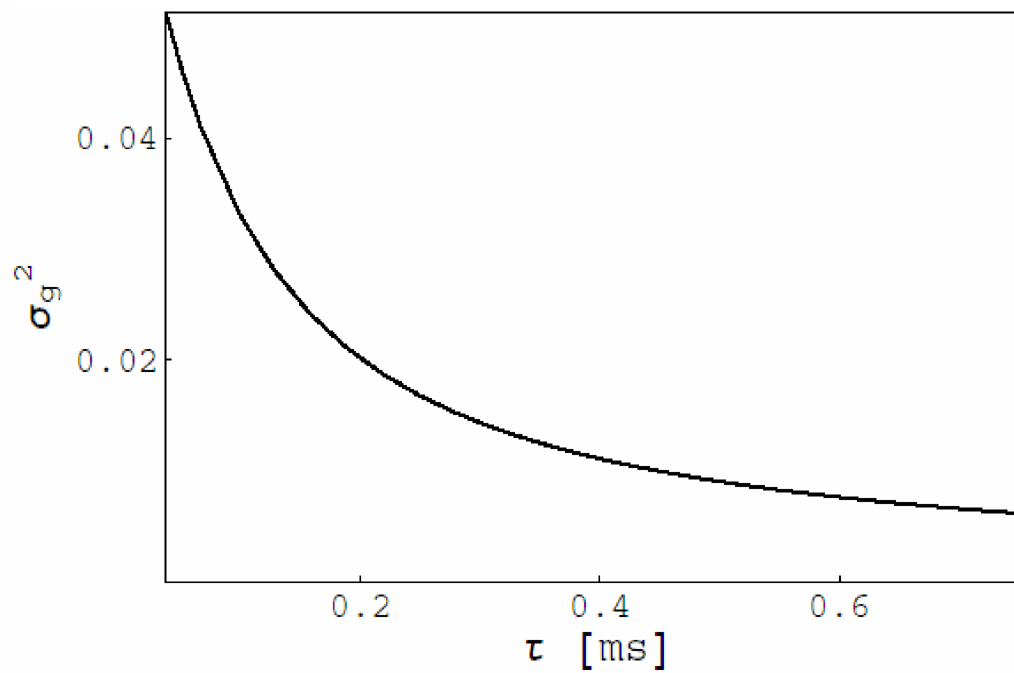


S. Filipp, Y. Hasegawa, R. Loidl and H. Rauch, Phys.Rev. A72 (2005) 021602

Future Prospects: Decoherence

[De Chiara and Palma, PRL 91, 090404 (2003)]





Variance of geometric phase (σ_g^2) tends to 0 for increasing time of evolution in magnetic field.

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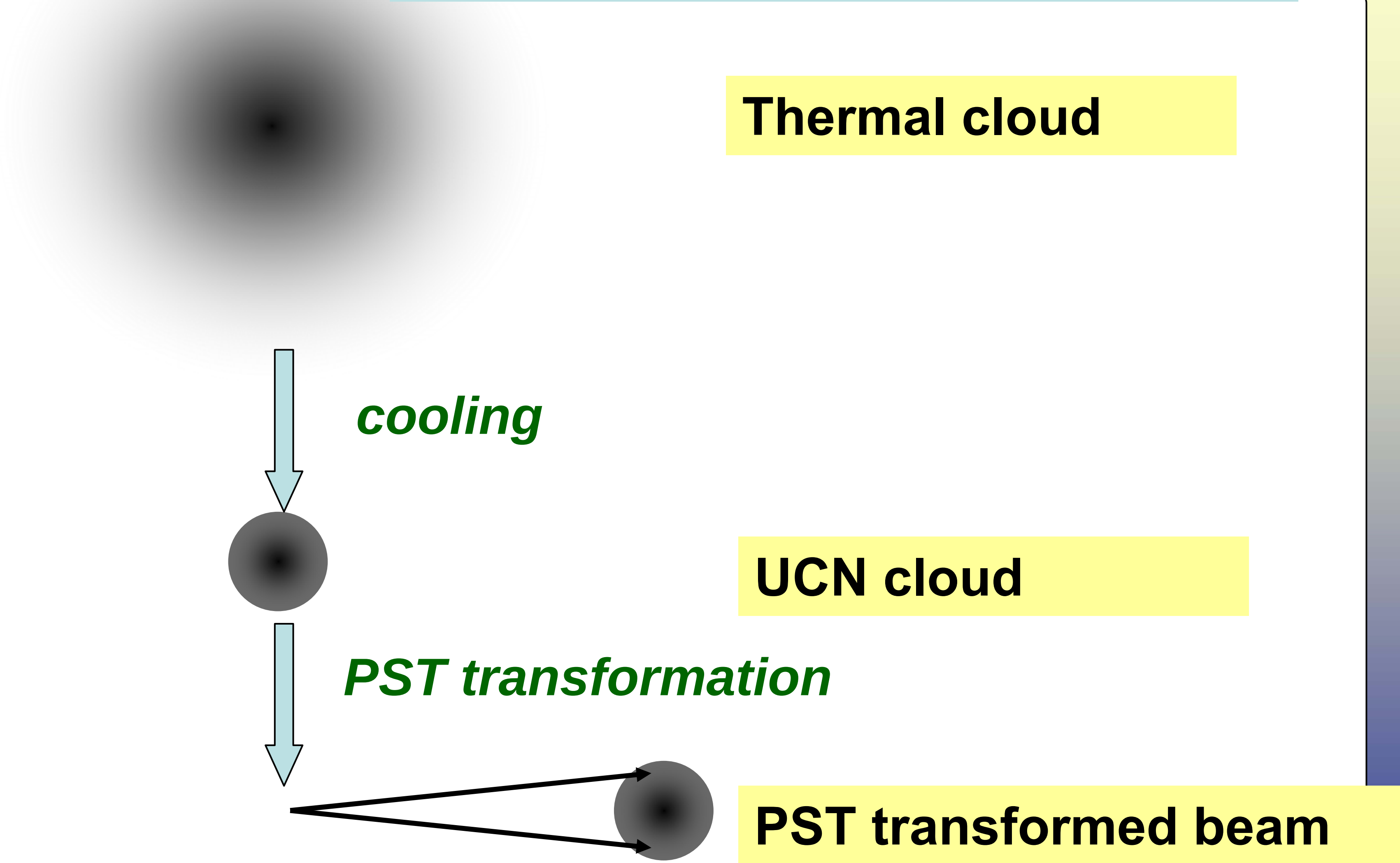
- Ultracold Neutrons and
- Phase Space Transformation

UCN – PST beam tailoring



$$dN = w(\vec{x},\vec{k})N\,d\vec{x}\,d\vec{k}$$

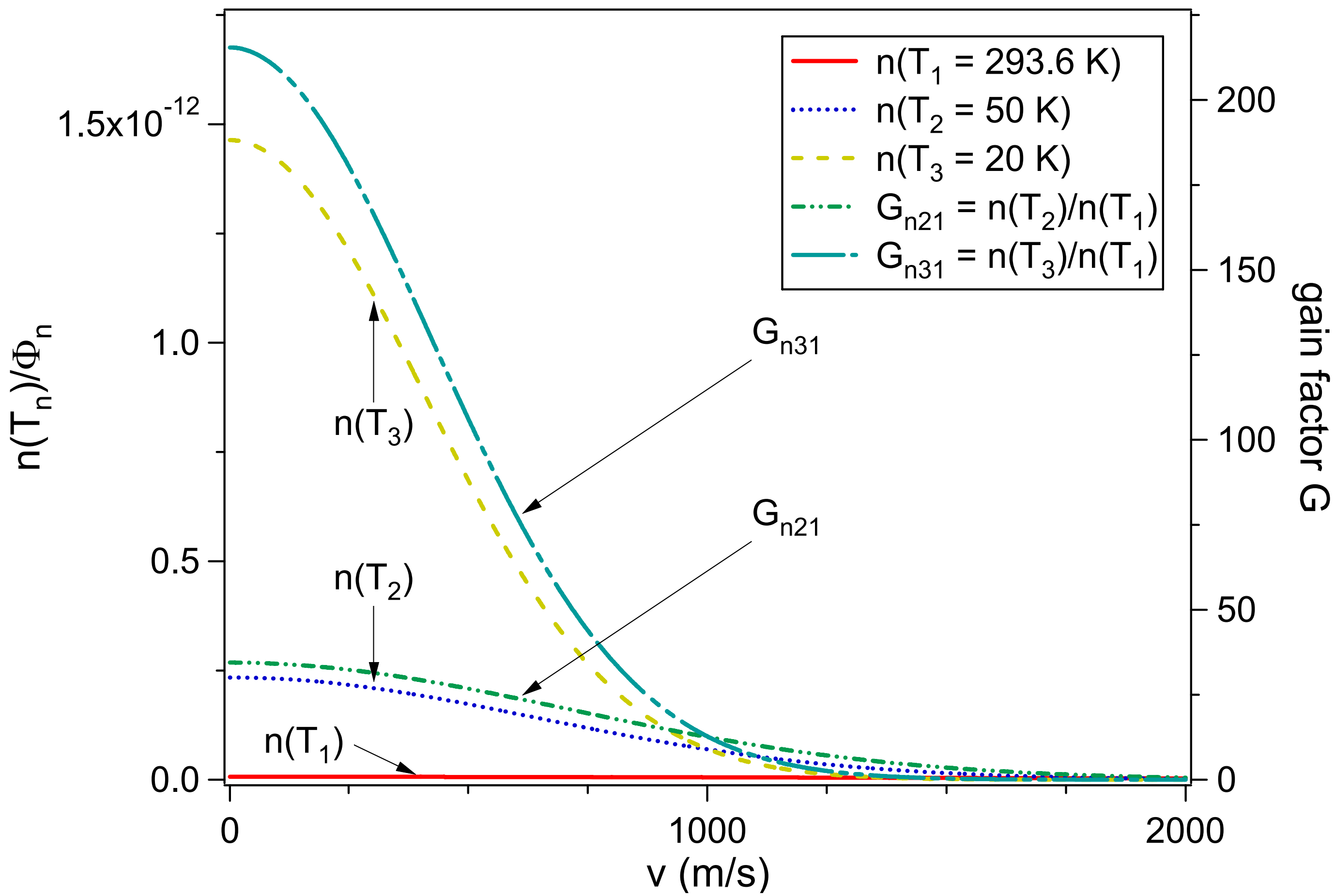
$$dN = \frac{1}{\pi^{3/2}k_T^3}N\,e^{-\frac{\hbar^2k^2}{2mk_BT}}\,d\vec{x}\,d\vec{k}$$

$$\Phi = \frac{2N}{\sqrt{\pi}}v_T$$

$$dN = \frac{\Phi}{2\pi v_T\,k_T^3}e^{-\frac{\hbar^2k^2}{2mk_BT}}\,d\vec{x}\,d\vec{k}\,,$$

Maier-Leibnitz Formel

PHASE SPASE DENSITY AND GAIN FACTORS



$$n(T) = \frac{\rho(v,T)}{4\pi} = \frac{\Phi_n}{2\pi v_T^4} \exp\left(-\frac{v^2}{v_T^2}\right)$$

$$G(T_1,T_2) = \frac{n(T_2)}{n(T_1)} = \frac{v_{T_1}^4}{v_{T_2}^4} \exp\left[-v^2\left(\frac{1}{v_{T_2}^4} - \frac{1}{v_{T_1}^4}\right)\right]$$

LUMINOSITY

$$L_z(T) = n(T)\,\frac{\mathrm{d}V_P}{\mathrm{d}t\,\mathrm{d}A\,\mathrm{d}\Omega} = \frac{\Phi_n}{2\pi}\,\frac{v_z\,v^2}{v_T^4}\,\exp\left(-\frac{v^2}{v_T^2}\right)\mathrm{d}v_z$$

$$\mathrm{d}V_P = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}v_x\,\mathrm{d}v_y\,\mathrm{d}v_z$$

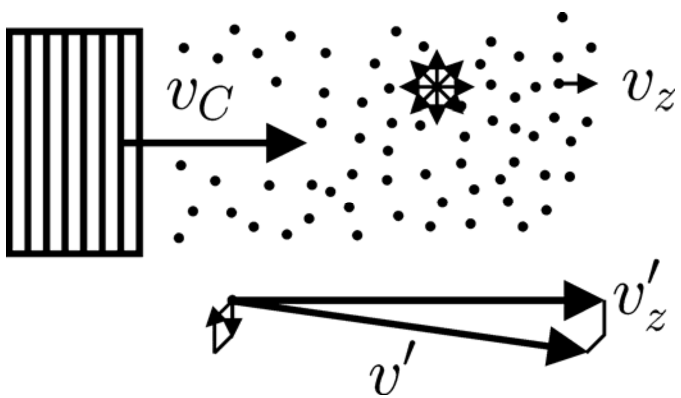
$$dt = \frac{\mathrm{d}z}{v_z}$$

$$dA = dxdy$$

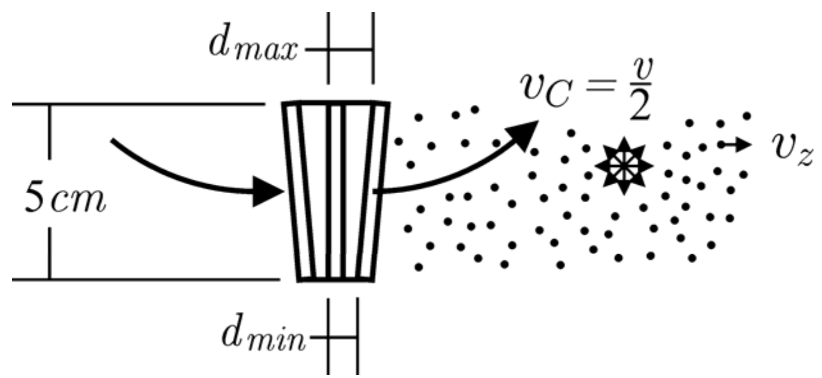
$$d\Omega = dv_x dv_y / v^2$$

Transformer types

linear



rotating

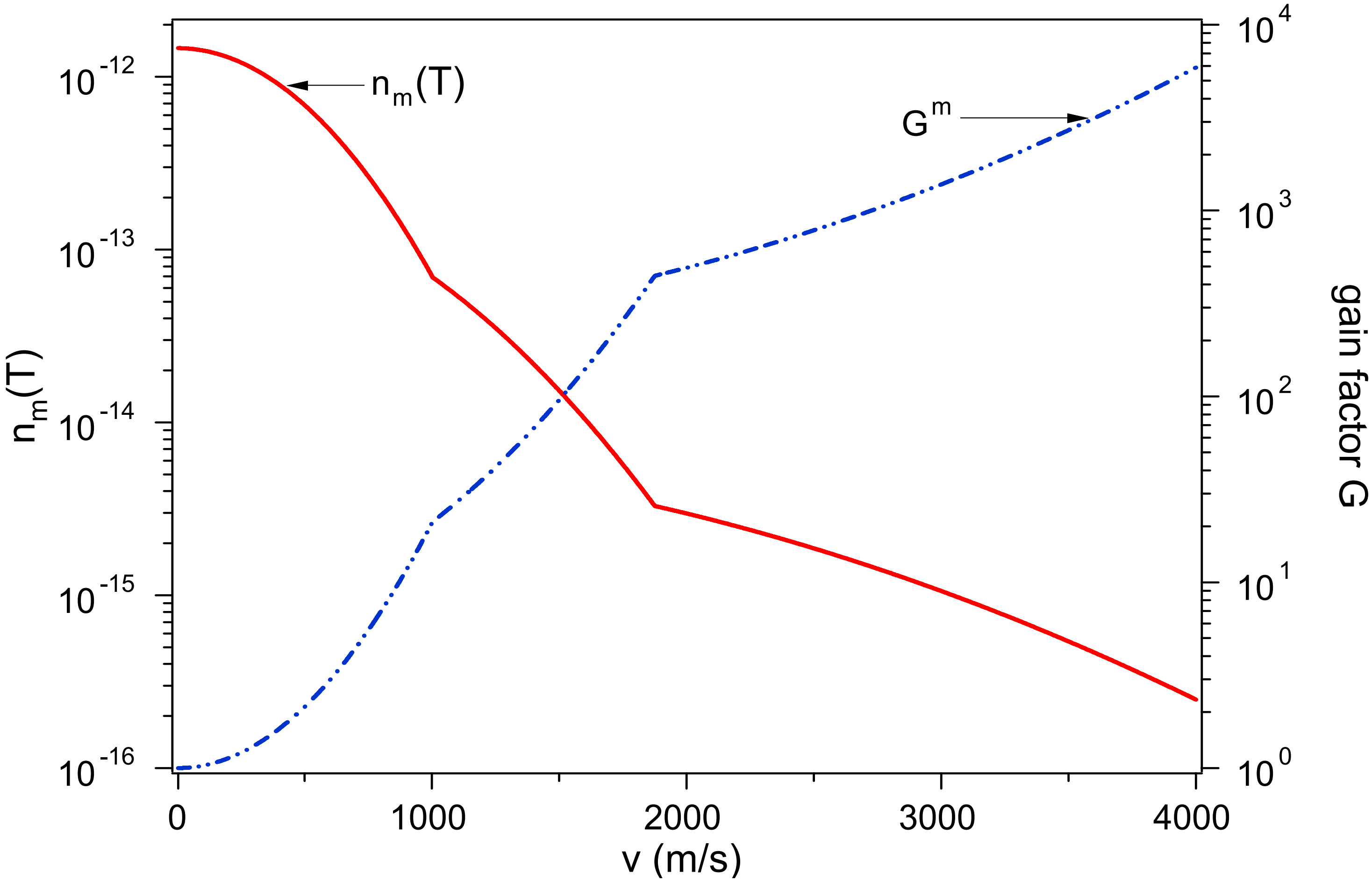


$$v_c = 2\pi r\nu = h/md_{hkl}$$

Example: r =35 cm, ν = 13,648 rpm

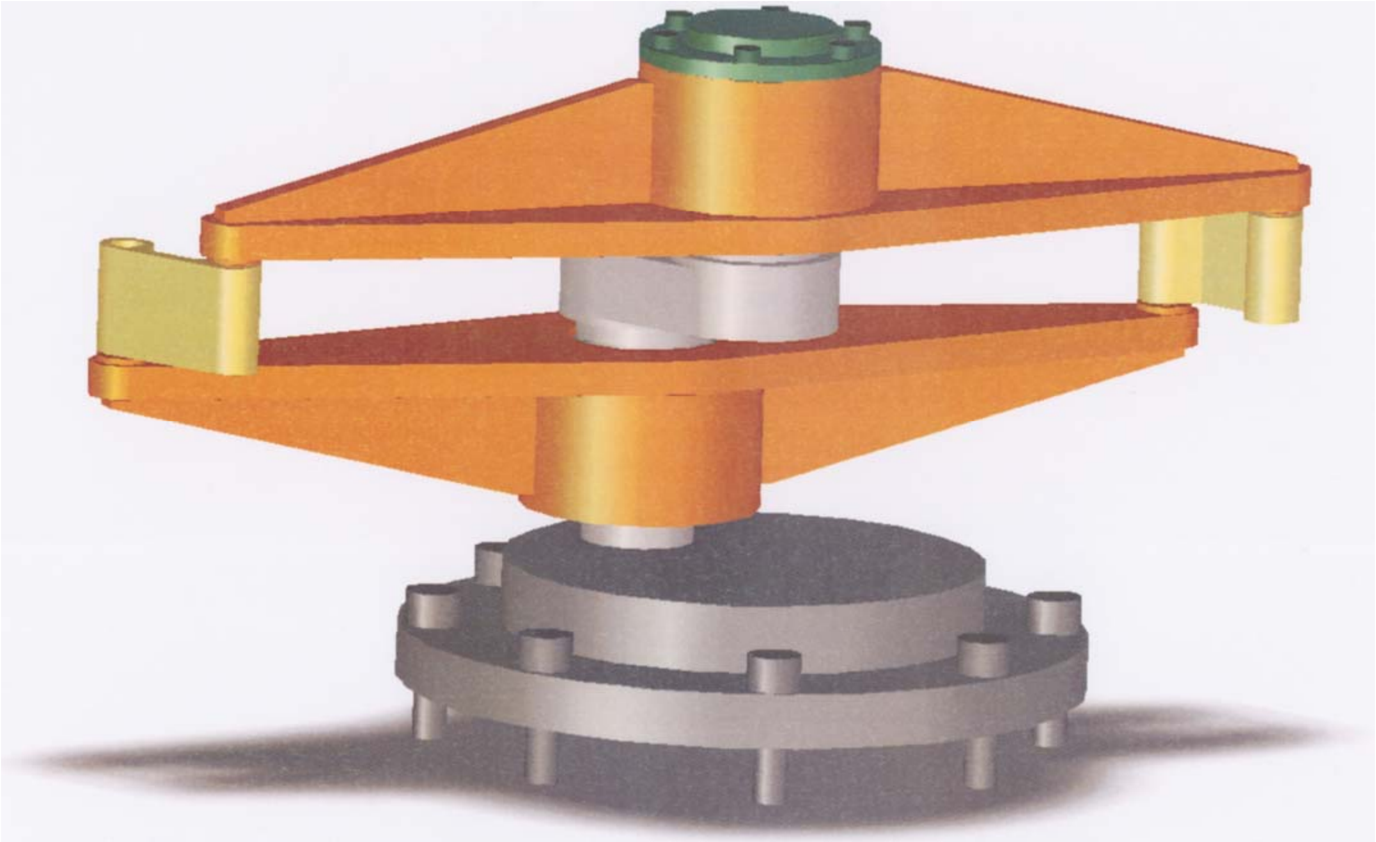
⟶ v_c = 500 m/s (v = 1000 m/s)

d_0 = 4.0 A, $\Delta d/d_0$ = 6%



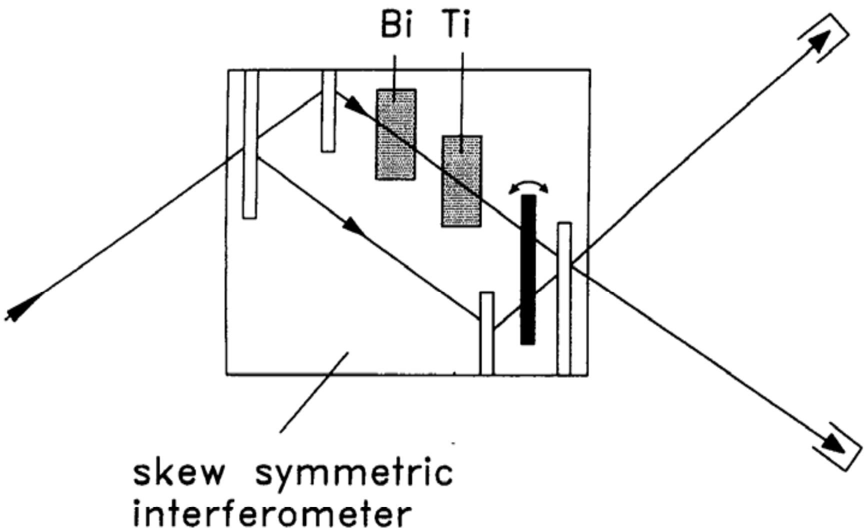
$$G_S^m = \frac{n_S(T_3)}{\max\{n(T_1), n(T_2), n(T_3)\}}$$

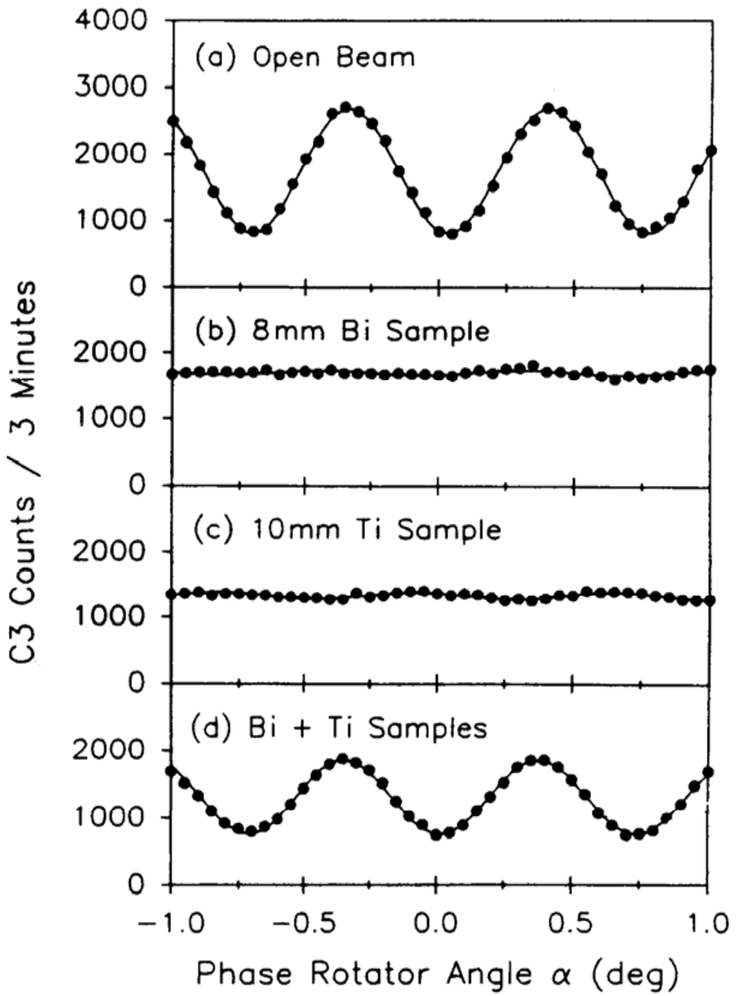
Double rotator system



Thank You

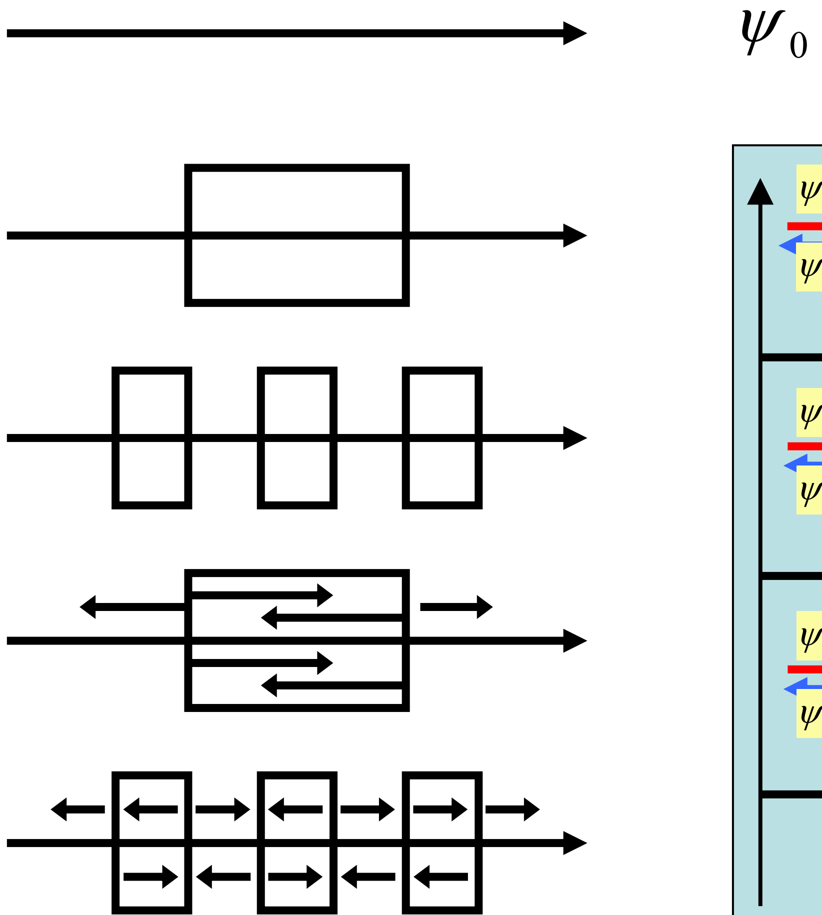
Phase Echo

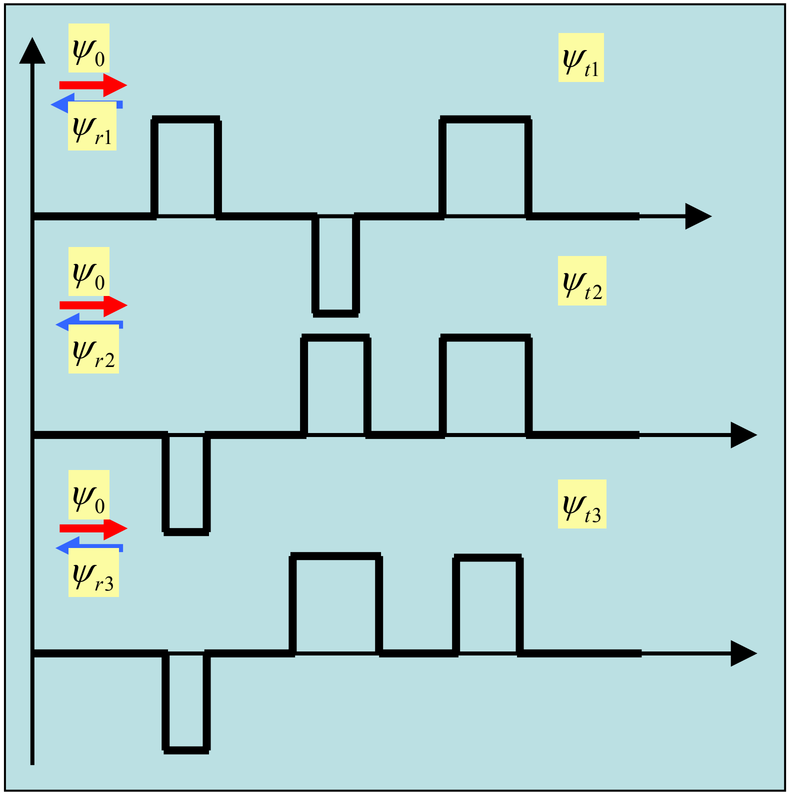




Clothier R., Kaiser H., Werner S.A., Rauch H., Wölwitsch H., Phys.Rev.A44 (1991)5357

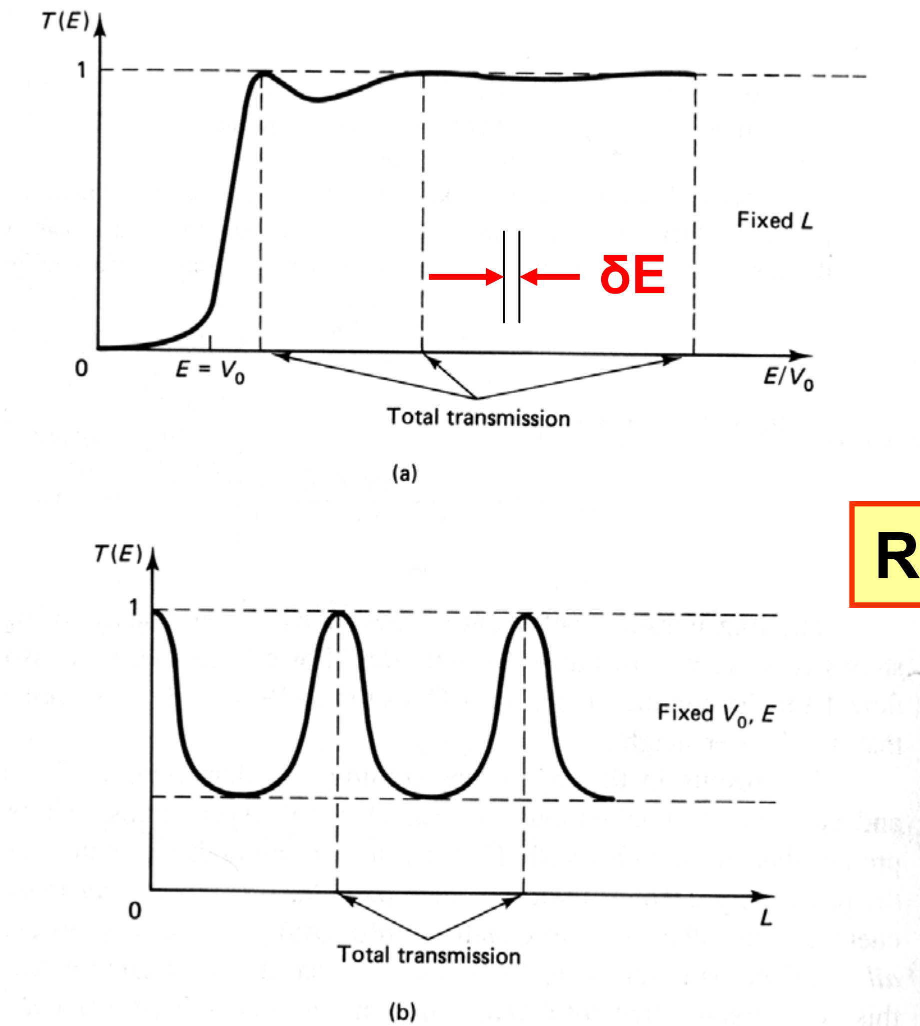
Reversibility-Irreversibility





Source) → ⟶⟵ **(Detector**

Barrier Reflectivity



T + R = 1

→ **T < 1**

$$R_{Min} = (V/2E)^2 \delta k^2 L^2 > 0$$

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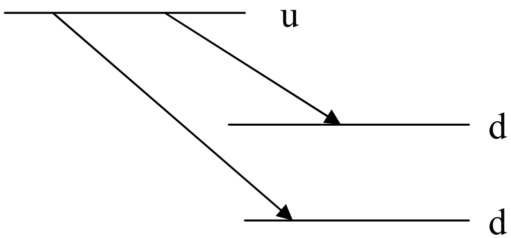
QUANTUM ZENO EFFECT

TU VIENNA

B. Misra and E.C.G. Sudarshan 1977



$$|\psi(t)\rangle = \exp(-iHt/\hbar)|u\rangle$$

$$= a_u(t)|u\rangle + \sum_i a_{d_i}(t)|d_i\rangle$$

Probability of finding the system undecayed:

$$P(t) = |a_u(t)|^2 = |\langle u|\exp(-Ht/\hbar|u\rangle|^2$$

$$\approx 1-(\Delta H/\hbar)^2 t^2 + O(t^4)\ldots$$

$$(\Delta H)^2 = \langle u|H^2|u\rangle - \langle u|H|u\rangle^2$$

Measurements: N-times in [0,t]

$$P_N(t) = \left[1-(\Delta H/\hbar)^2\left(\frac{t}{N}\right)^2\right]^N$$

$$N \to \infty$$

$$= 1-(\Delta H/\hbar)^2\left(\frac{t^2}{N}\right) + \ldots \to 1$$

a) Spin rotation

$$P_+ = \cos^2\left(\frac{\omega_L \ell_0}{2v}\right) \to 0$$

$$\ell_0 = (2m+1)\pi v/\omega_L$$

$$(\omega_L = 2|\mu|B/\hbar)$$

b) Zeno situation (ideal)

$$P_+ = \left[\cos^2\left(\frac{\omega_L \ell}{2v}\right)\right]^n = \left[\cos^2\frac{\pi}{2n}\right]^n \xrightarrow[n\to\infty]{} 1$$

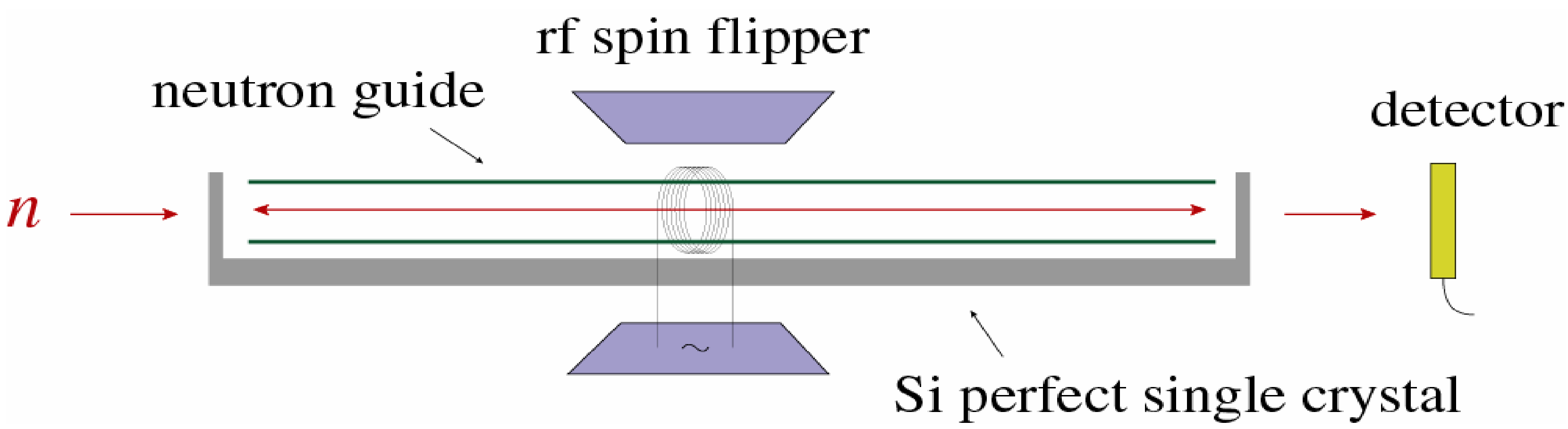
$$\ell = \ell_0/n$$

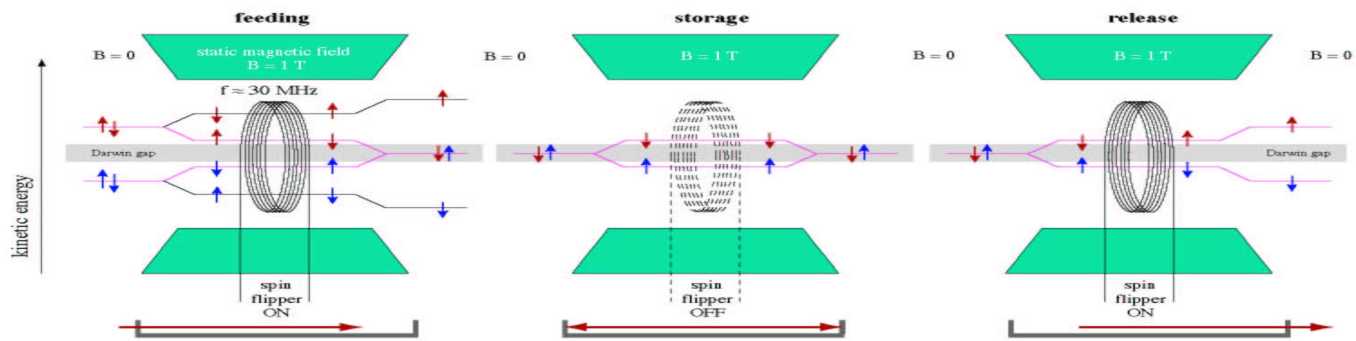
c) Zeno situation (real)

$$I_+ = P_+\overline{T}^n = P_+(1-\overline{R})^n = P_+\left[1-\frac{1}{2}\left(\frac{V_0}{2E}\right)\right]^n$$

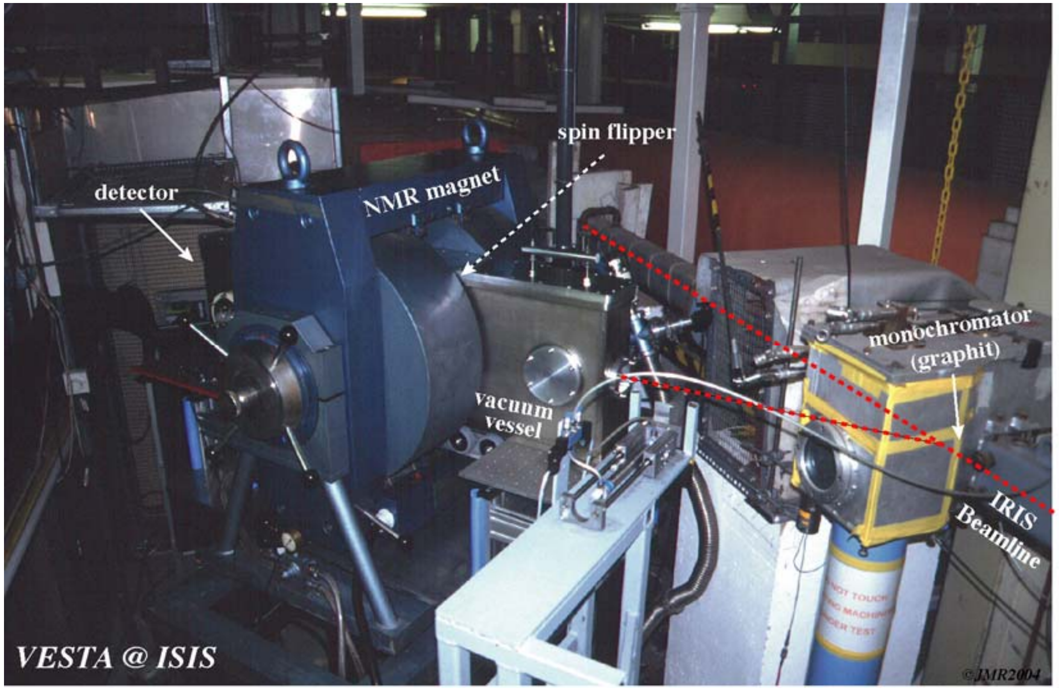
$$\cong 1-\frac{n}{2}\left(\frac{V_0}{2E}\right)^2 + \ldots \xrightarrow[n\to\infty]{} 0$$

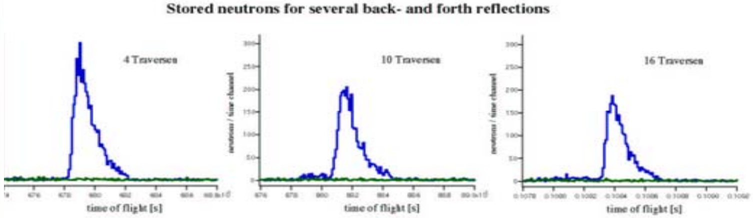
ISIS - Resonator - Principles

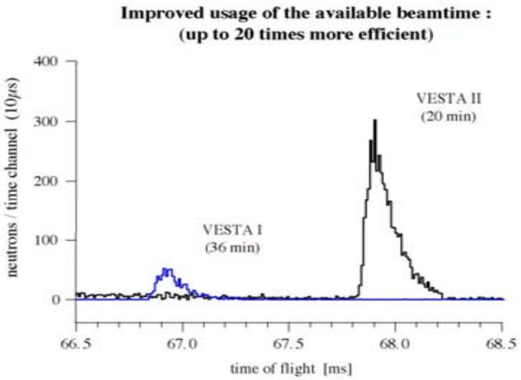




Set-up at ISIS Spallation Source



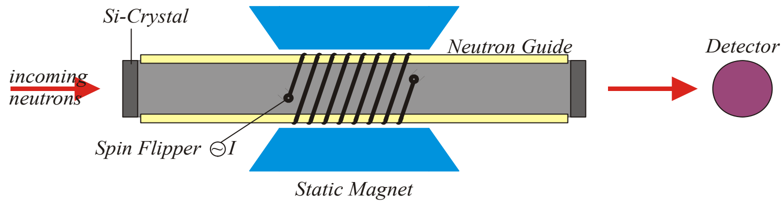


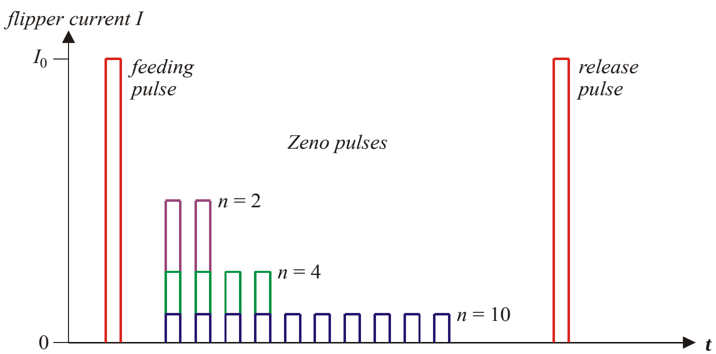


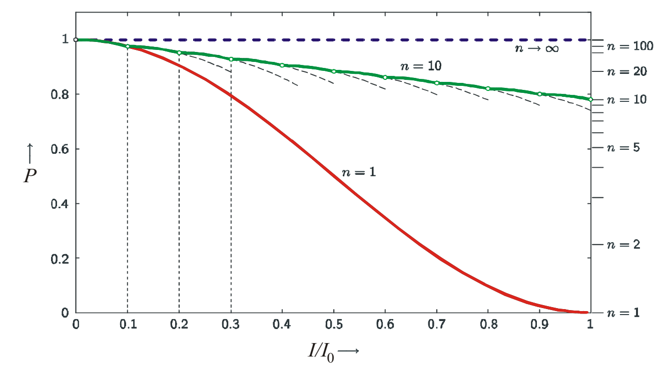
M.R.Jaekel, E.Jericha, H.Rauch, Nucl.Instr.Meth. A539 (2005) 335

Quantum Zeno Effect

$$P_{Survival} = \cos^2\left(\frac{\mu B l}{\hbar v}\right) = \cos^2\left(\frac{\pi I}{2I_0}\right) \xrightarrow[I \to I_0]{} 0$$





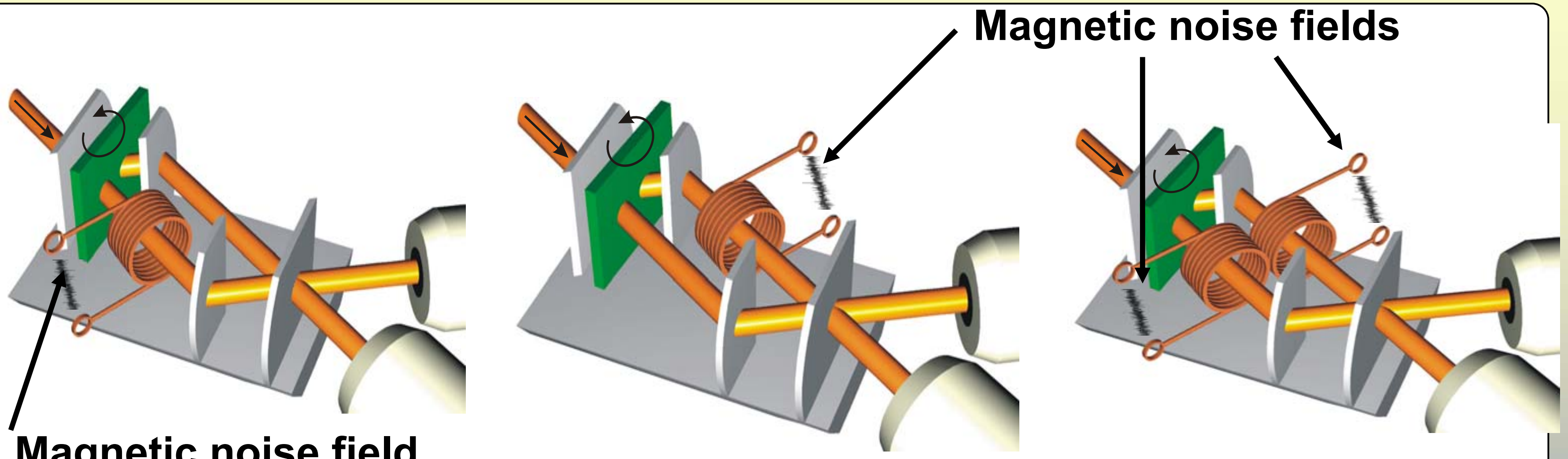


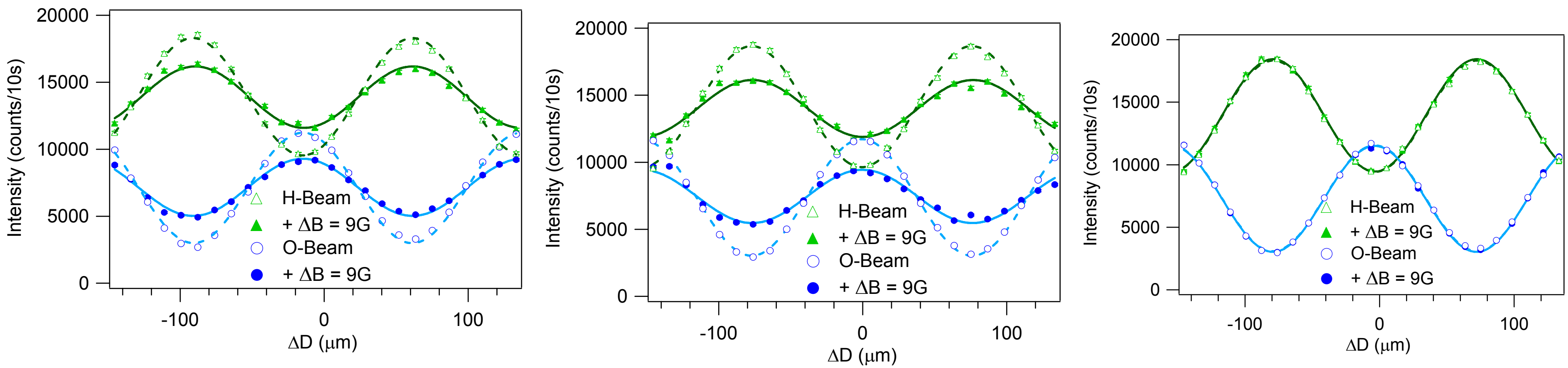
E.Jericha, M.Jaekel, S.Pascazio, H.Rauch (in progress)

Summary

- *The neutron is an ideal tool for quantum experiments*
- *Quantum mechanics has been verified illustrating some of its strange features*
- *There is no natural limit between quantum and classical world*
- *Non-locality and contextuality are fundamental laws*
- *There is much more information in a quantum system than usually extracted*
- *Unavoidable quantum losses may play an important role in the understanding of decoherence phenomena*

Dephasing at low order





M.Baron, H.Rauch, M.Suda (in progress)

ATOMINSTITUT

Quantum Properties of Neutrons

TU VIENNA

Helmut Rauch

Atominstitut der Österreichischen Universitäten, Wien

Remarks on Neutron Interferometry

Quantum State Preparation and Measurement

Magnetic Noise Dephasing

Confinement induced phase

Contextuality

Quantum State Tomography and Topological Phases

Unavoidable Losses

Contents – Chapter

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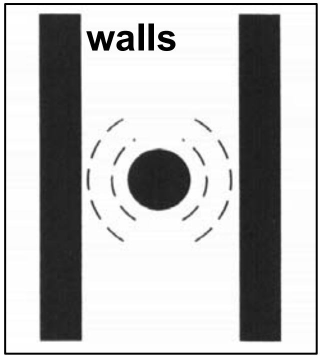
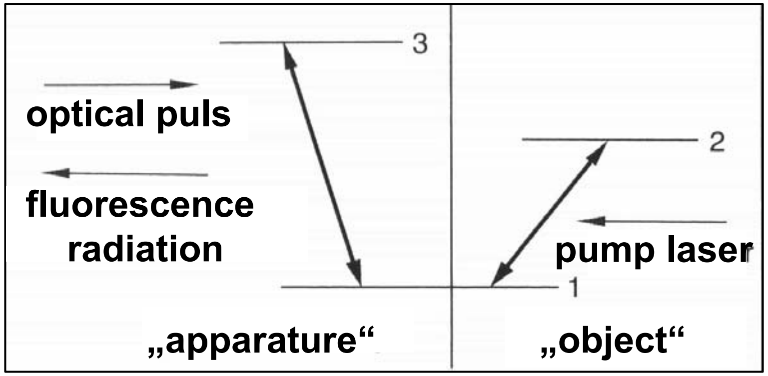
Confinement effects

Spatial Confinement (Casimir-Effect)



Decay depends on
Distance

Temporal Confinement (Zeno-Effect)

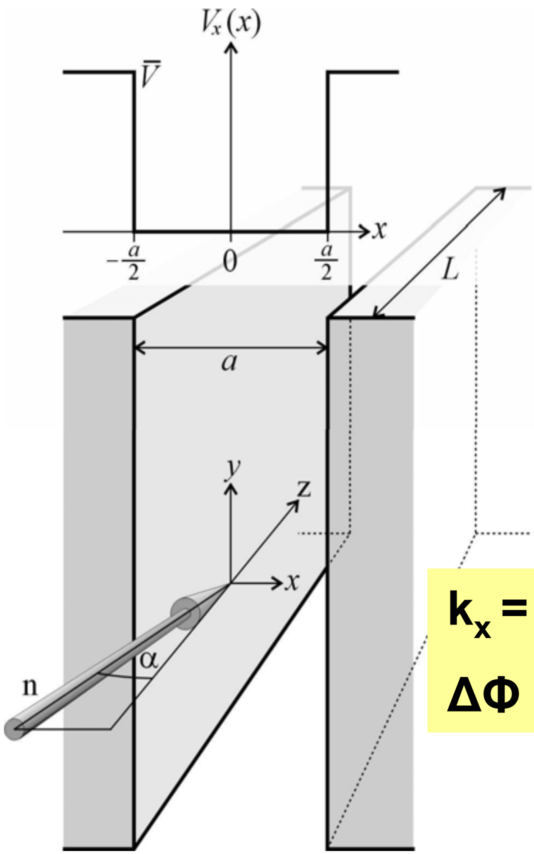


Decay depends on
Observation Frequency

Neutrons within a Slit

Proposal by: J.M.Levy-Leblond 1987; D.M.Greenberger 1988

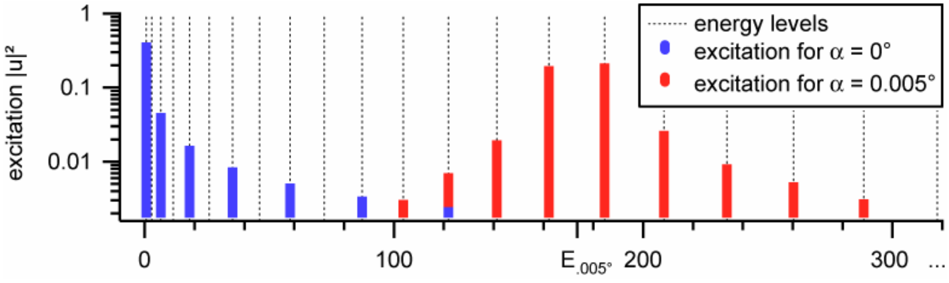
- **187 levels**
- $E_0 = 0.4872$ **peV**



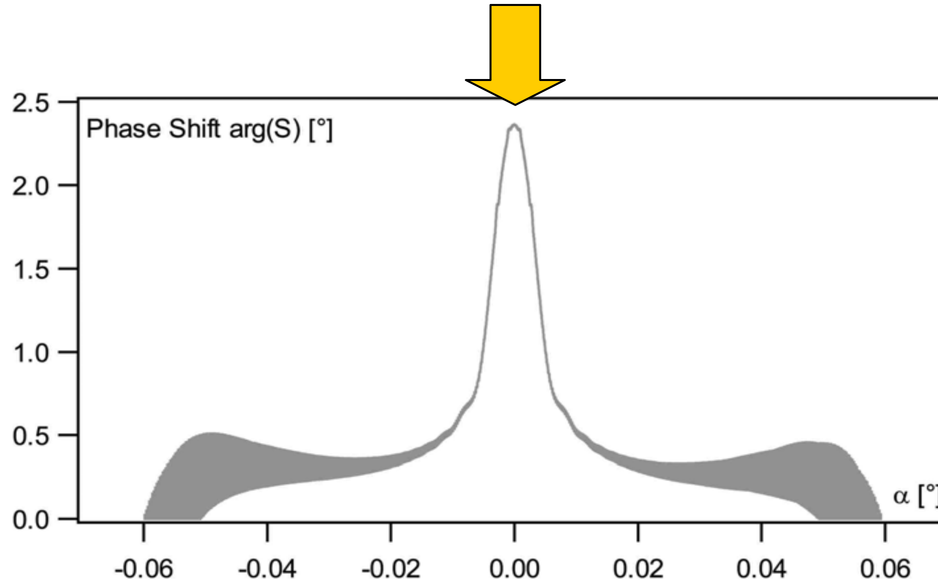
$k_x = n\pi/a$

$\Delta\Phi = l.\Delta k_z$

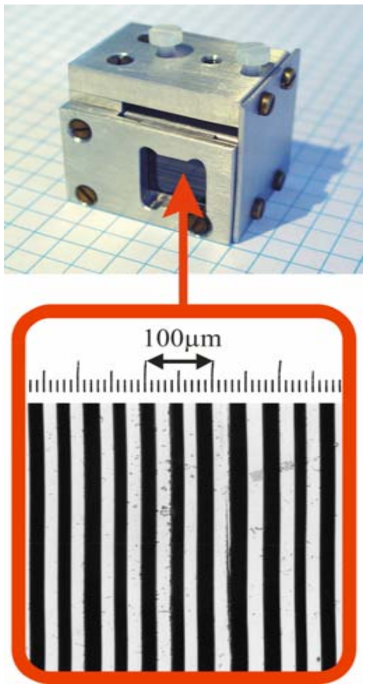
Level excitation



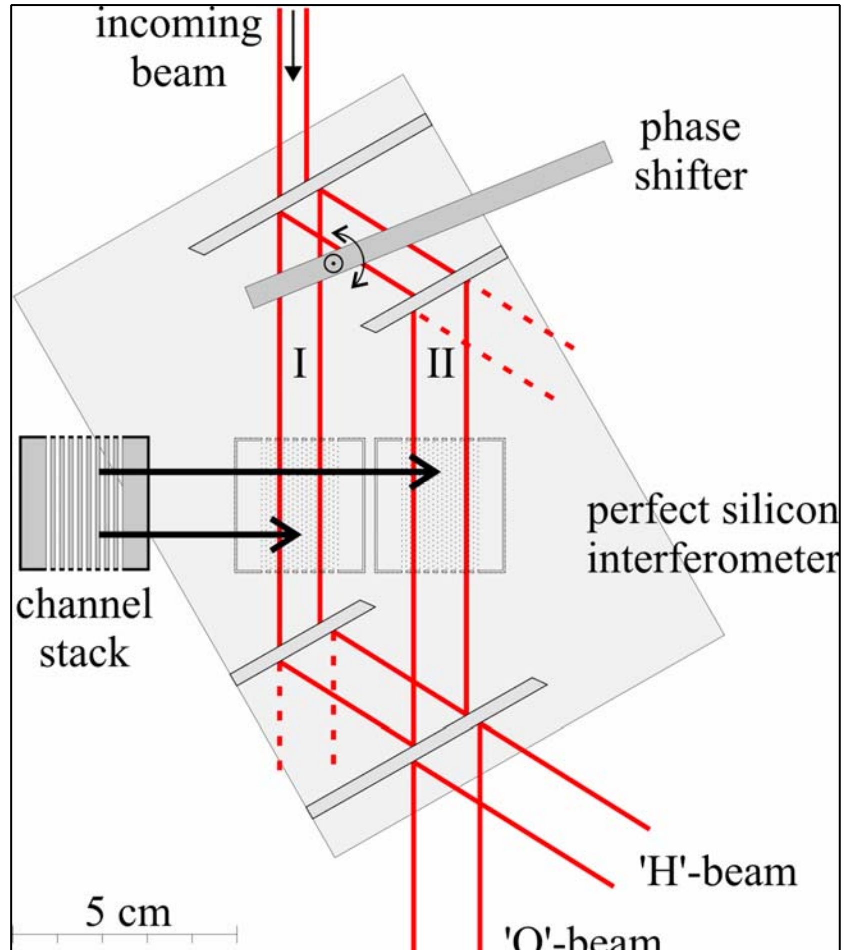
a =22.1 ± 3.1μm



Confinement Induced Phase



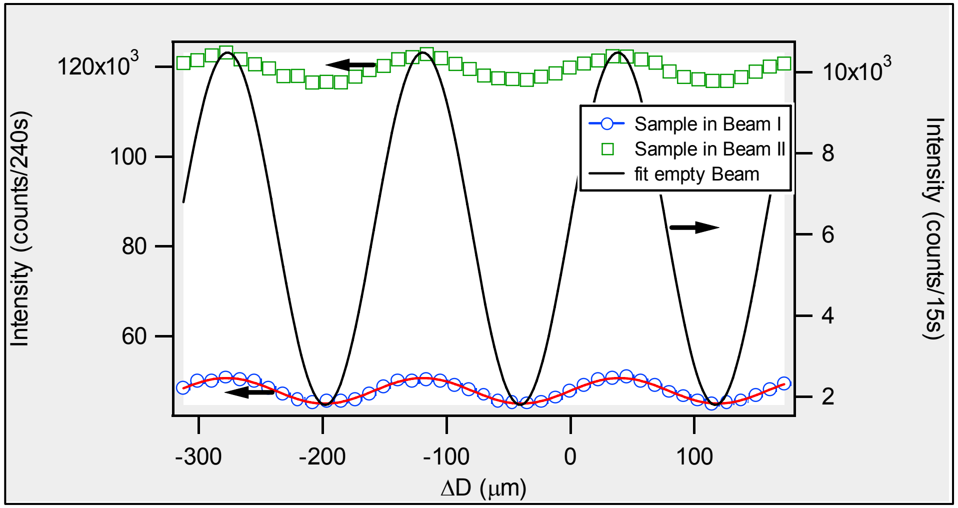
Experimental Setup



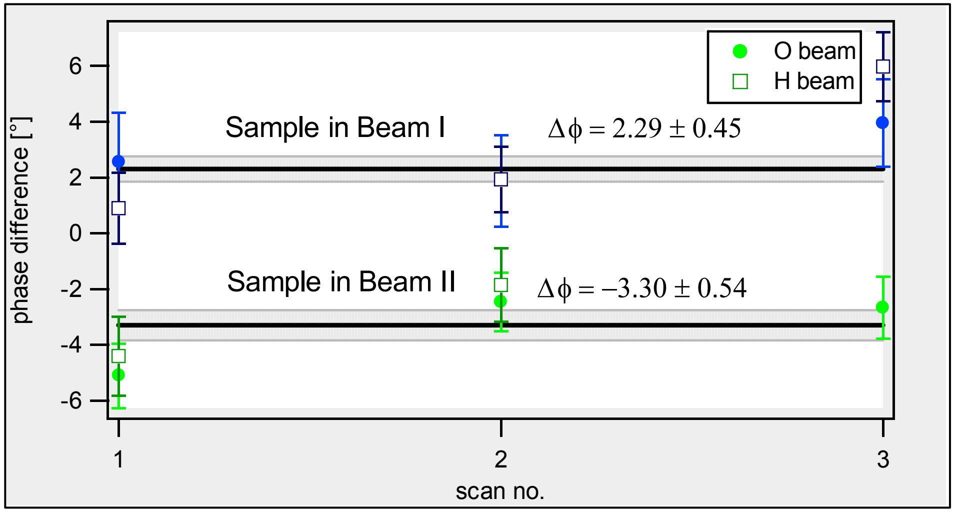
Calc. ΔΦ = 2.50°
Exp. ΔΦ = 2.8(4)°

H.Rauch, H.Lemmel, M.Baron, R.Loidl, Nature 417 (2002) 630

Measured Phase Shift and Results



Measured Interference Pattern



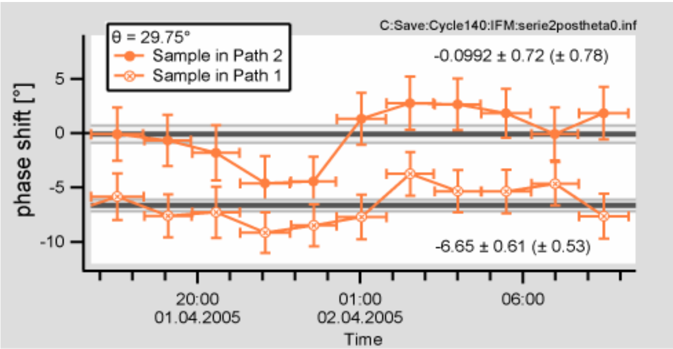
Summary of Measured Phase Shifts

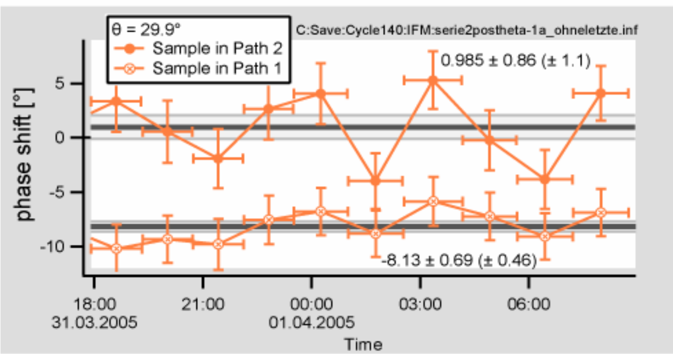
Calc. $\Delta\Phi = 2.5^0$

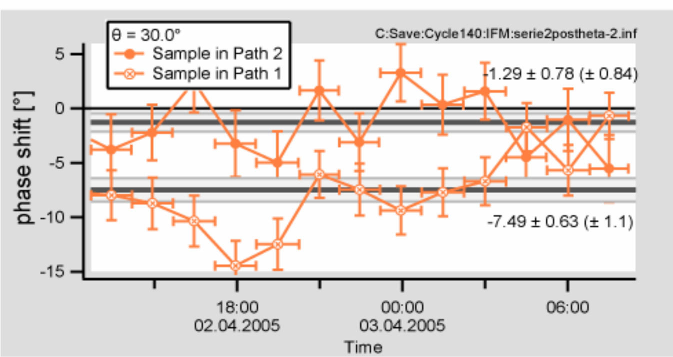
Exp. $\Delta\Phi = 2.8(4)^0$

H.Rauch, H.Lemmel, B.Baron, R.Lemmel; Nature 417 (2002) 630

New Measurements

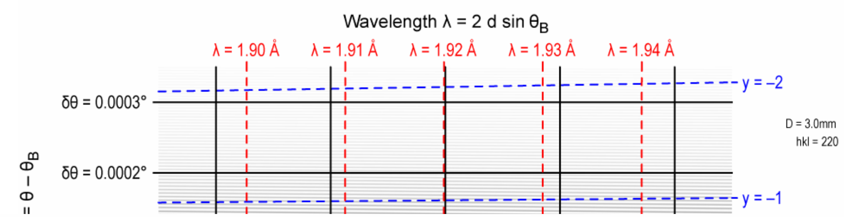


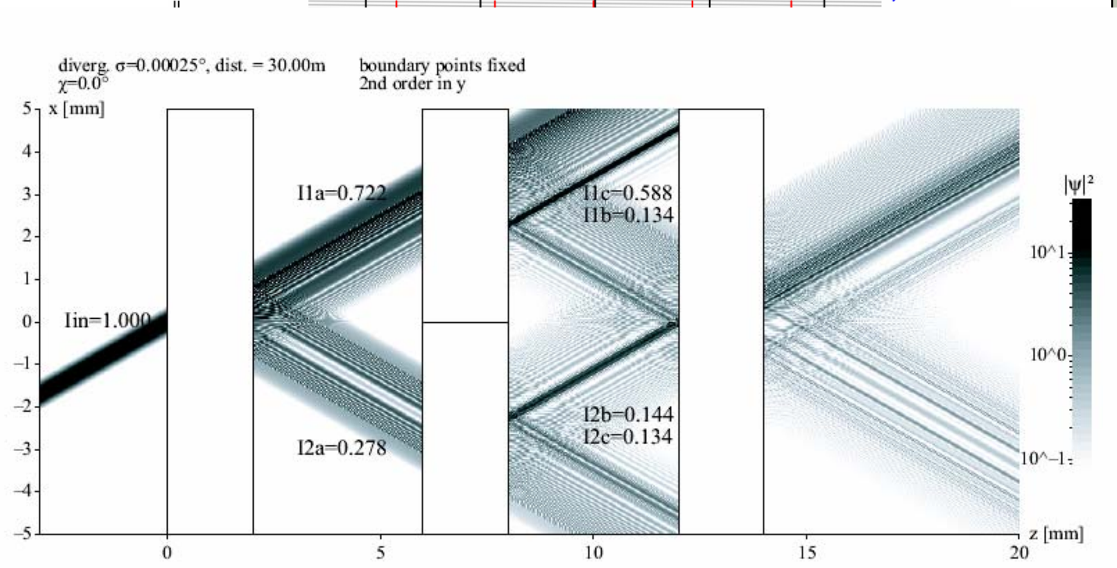




Wave field inside slits

Beam Intensity after Two Laue Reflections





H.Lemmel, R.Loidl, H.Rauch (in progress)

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- Remarks on Neutron Interferometry
- Quantum State Preparation and Measurements
- Magnetic Noise Dephasing
- Confinement induced phase
- **Contextuality**
- Quantum State Tomography
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Remarks on Neutron Interferometry

Quantum State Preparation and Measurements

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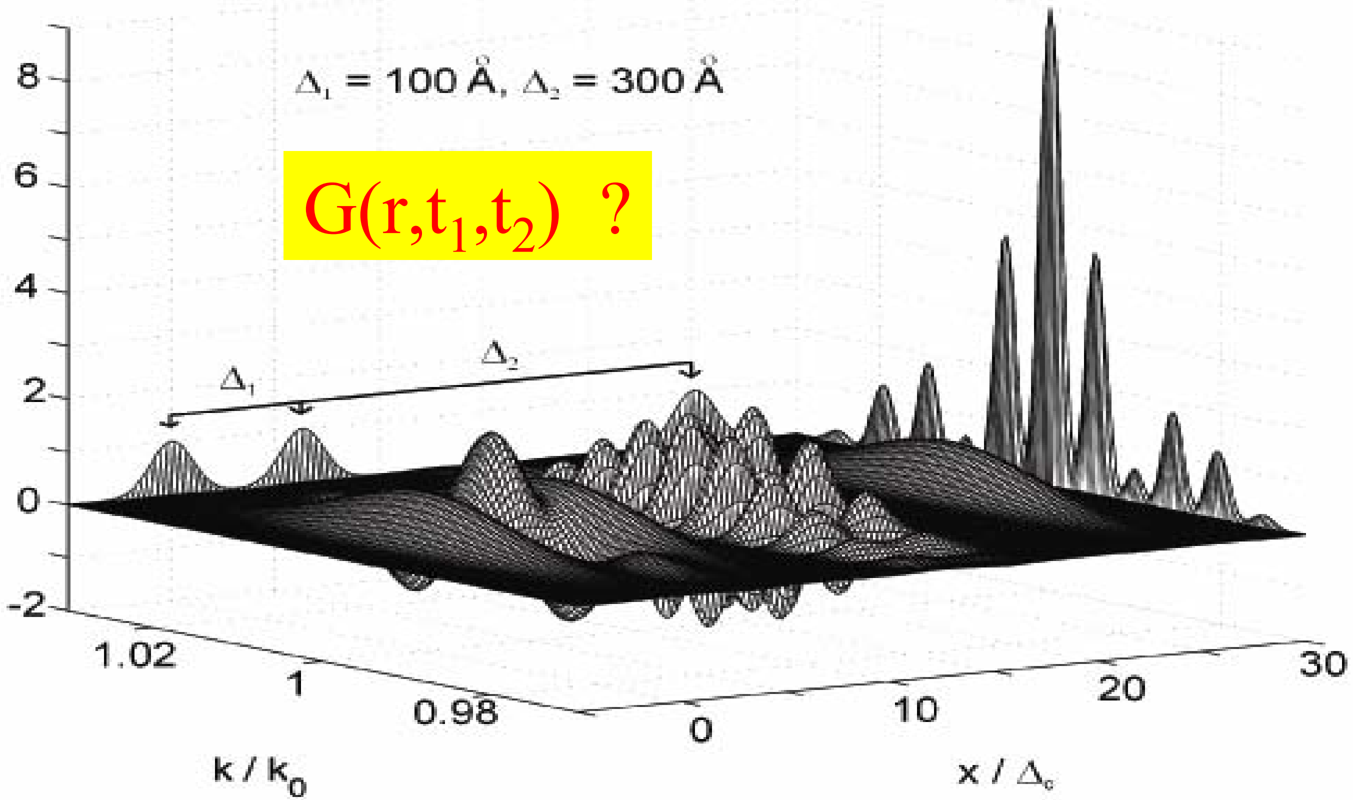
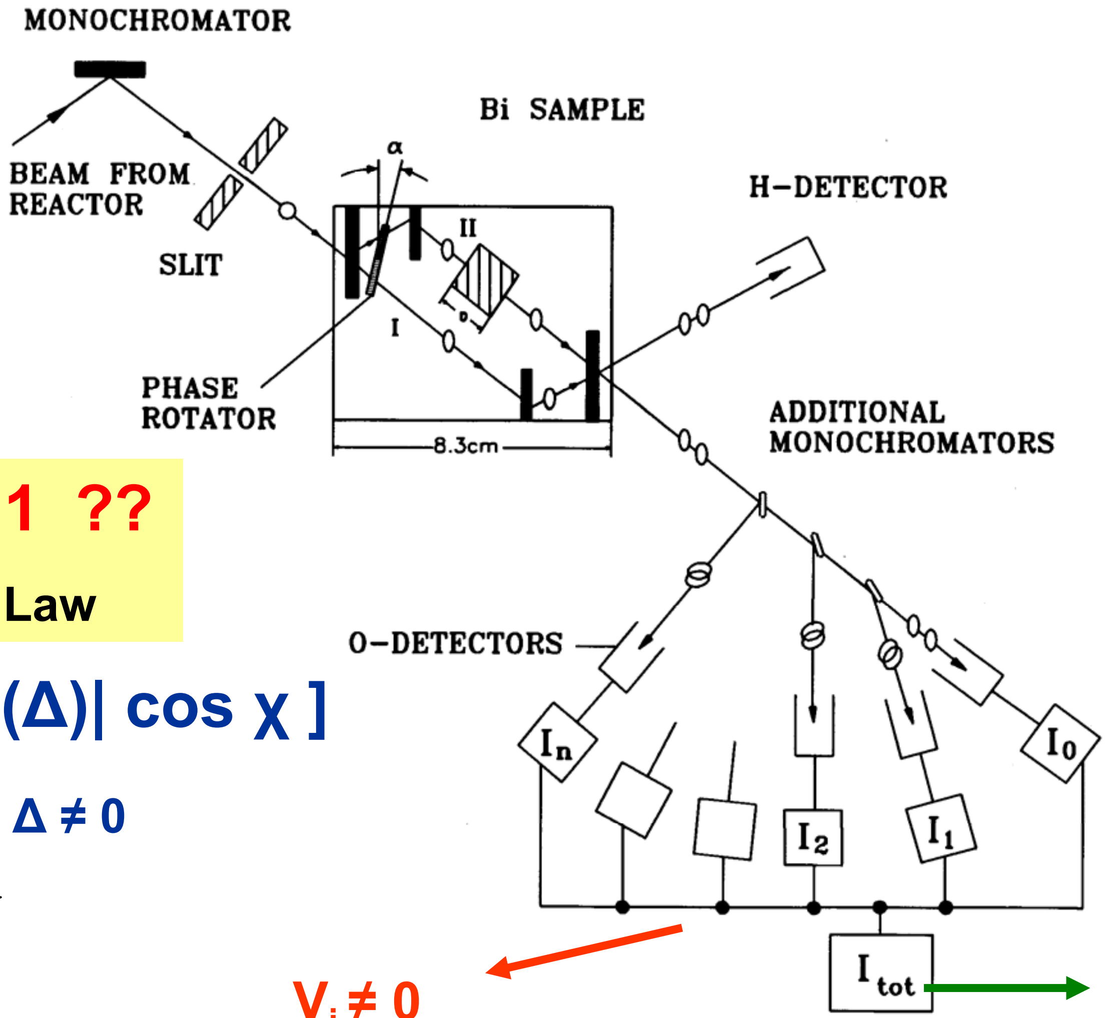
TRIPLE PEAK WIGNER FUNCTION



H.Rauch, M.Suda 2001

Momentum Postselection



$P^2 + V^2 \leq 1$??

Englert's Law

$$I = A\,[\,1 + |\Gamma(\Delta)|\,\cos\chi\,]$$

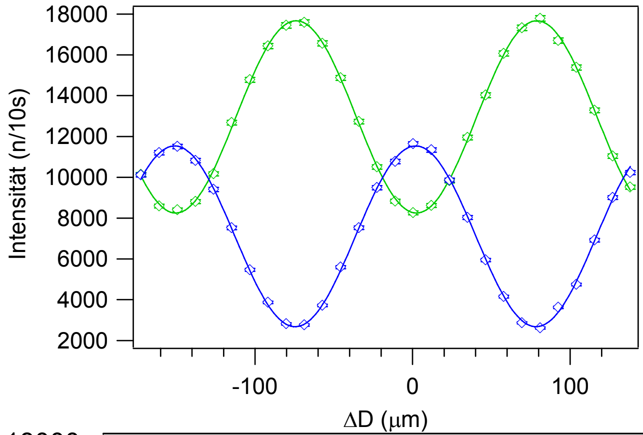
$|\Gamma(\Delta)| < 1$ for $\Delta \neq 0$

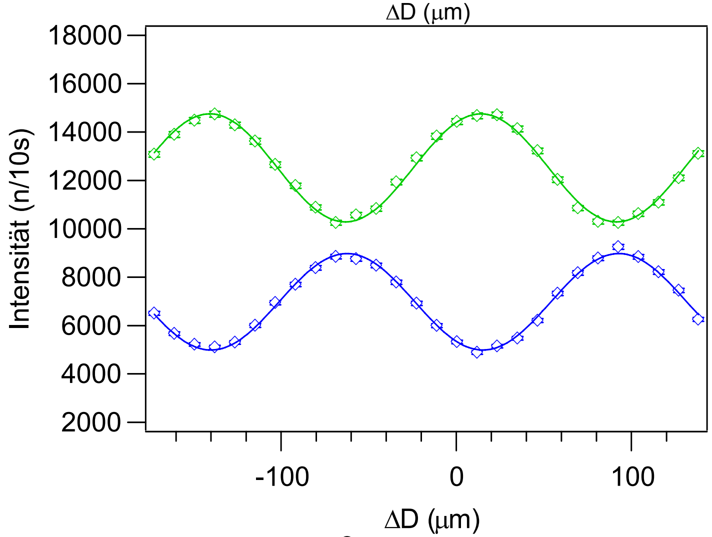
$V < 1$

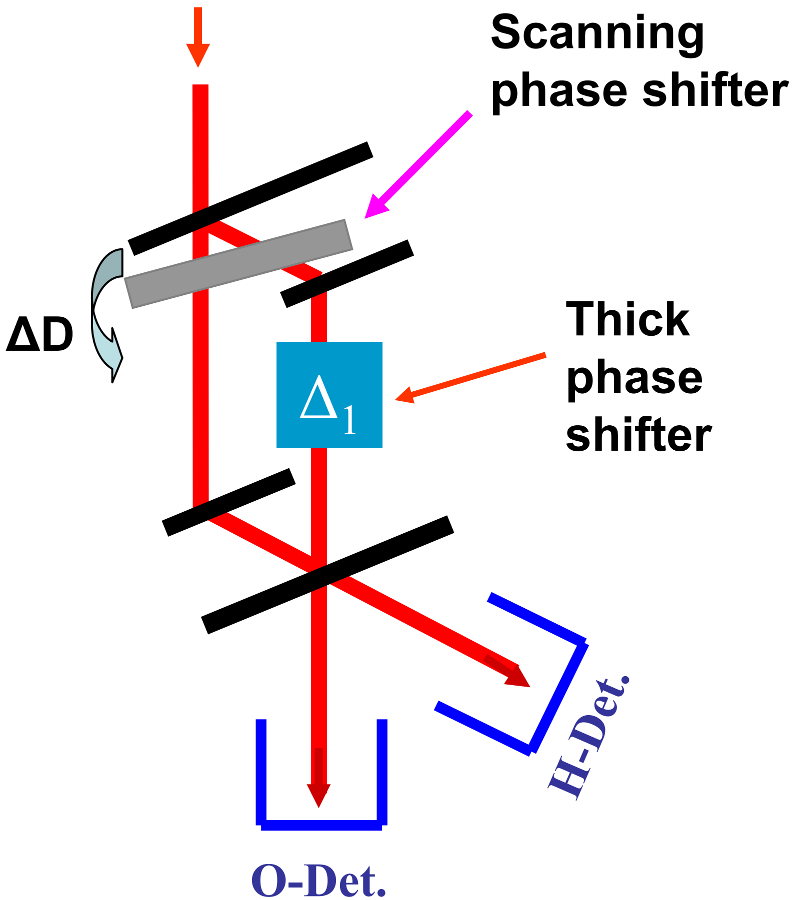
$V_i \neq 0$

$V_{tot} = 0$

Interference pattern measurements







$$I_O \propto \left|\psi_O{}^{I} + \psi_O{}^{II}\right|^2 \propto A + B\cos\chi$$

$$\chi = \oint \vec{k}\, d\vec{s} = (1-n)kD_{eff} \equiv -Nb_c\lambda D_{eff} = \Delta_1 \cdot k$$

Spectrum Modulation